Theoretical and Experimental Analysis of MEMS Ohmic Switch Design and Control for Reliability Improvement

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CORK INSTITUTE OF TECHNOLOGY

Theoretical and Experimental Analysis of MEMS Ohmic Switch Design and Control for Reliability Improvement

CUONG DO

2012
Theoretical and Experimental Analysis of MEMS Ohmic Switch Design and Control for Reliability Improvement

by

Cuong Do, M.Eng.

Nimbus Centre for Embedded Systems Research
Dept. of Electronic Engineering, Cork Institute of Technology

Supervised by Dr. Martin Hill & Dr. Maryna Lishchynska

Submitted in Partial Fulfilment of the Requirements of

Doctor of Philosophy

to Cork Institute of Technology, June 2012
Declaration

I hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person nor material to a substantial extent has been accepted for the award of any other degree or diploma of the university of higher learning, except where due acknowledgement has been made in the text.

Signature of Author: [Redacted]

Certified by: [Redacted]

Date: 4th Sept. 2012
I dedicate this Dissertation to my loving parents
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List of Publications


MEMS devices are micromechanical components fabricated using semiconductor fabrication technologies. With advanced functionalities and miniaturization, MEMS devices are being employed in a wide variety of applications, especially in mobile networks. The MEMS ohmic switch is one of the most promising MEMS devices and has the same principle of a mechanical moving switch to manipulate an electrical signal. Due to the existence of mechanical movement and its compactness, reliability issues have been observed as the main barrier to commercialisation.

This thesis addresses the key issue of reliability in the rapidly expanding area of MEMS. In particular MEMS ohmic switches are considered. The reliability issues are broad and this thesis does not have the ambition to cover all reliability aspects of this type of device. The thesis focuses on identifying operational reliability issues, proposing models for design optimisation and control methods to reduce factors that affect the lifetime of the devices. There are 3 main contributions to the field.

Firstly, this thesis presents a new generalized analytical method for determining the pull-in instability of cantilever beams subjected to partial electrostatic load. It employs a non-linear stiffness analysis. The method is then developed to achieve an analytical design guide to avoid pull-in instability issues, thus reducing the impact force of contact, of cantilever beams. The difference between the developed models and Finite Element Model (FEM) are less than 3%.

Secondly, the integration of nonlinear contact mechanics with adhesion into a dynamic 2-D model is outlined. The contact resistance and switch
bouncing effects can be captured with high accuracy with experiment results. The developed model is helpful for evaluating the effect of materials and designs on the dynamic behaviour of the switch. The developed model is then employed to evaluate the switch degradation effects to predict the life-time of the switches.

Finally, the thesis introduces a novel energy-based approach to adaptive pulse shaping for control of MEMS ohmic switch closure. The method includes all of the most important practical effects. The method reduces the bouncing effect while maintaining fast switching. Experiment evaluation shows that the highest error is 6.5% at 45V input. Error reduces as the input voltage increases. The analytical method can be easily modified to adapt with the variance of system parameters drift during operation.

In this dissertation, the key failure mechanisms of the devices are identified. This thesis develops system models which allow for quick, high accuracy system evaluation. The models are used to improve the switch geometric design, to capture the dynamics of switch bouncing and to optimise the switch actuation voltage to eliminate contact bouncing. Through collaboration with our commercial partner the work can have an immediate benefit in improving switch reliability and therefore overcoming the principle barrier to commercialisation.
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<tr>
<td>CMOS</td>
<td>Complementary metal oxide semiconductor</td>
</tr>
<tr>
<td>2,3-D</td>
<td>Two, three-dimensional</td>
</tr>
<tr>
<td>CNT</td>
<td>Carbon nanotube</td>
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<tr>
<td>DC</td>
<td>Direct current</td>
</tr>
<tr>
<td>EMR</td>
<td>Electro Mechanical Relay</td>
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<tr>
<td>FDM</td>
<td>Finite Difference Method</td>
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<tr>
<td>FEM</td>
<td>Finite Element Method</td>
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<tr>
<td>FET</td>
<td>Field Effect Transistor</td>
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<tr>
<td>IC</td>
<td>Integrated Circuit</td>
</tr>
<tr>
<td>IIP3</td>
<td>Third-order intercept point</td>
</tr>
<tr>
<td>MEMS</td>
<td>Micro Electro Mechanical Systems</td>
</tr>
<tr>
<td>LDMOS</td>
<td>Laterally Diffused Metal Oxide Semiconductor</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>$S_{21}$</td>
<td>Transmission coefficient (dB)</td>
</tr>
<tr>
<td>SDOF</td>
<td>Single-degree-of-freedom</td>
</tr>
<tr>
<td>SEM</td>
<td>Scanning Electron Microscope</td>
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<tr>
<td>SOI</td>
<td>System-on-insulator</td>
</tr>
<tr>
<td>SOP</td>
<td>System-on-a-package</td>
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<tr>
<td>SPnT</td>
<td>Single Pole n Throw</td>
</tr>
<tr>
<td>SPST</td>
<td>Single Pole Single Throw</td>
</tr>
<tr>
<td>SS</td>
<td>Solid State</td>
</tr>
<tr>
<td>T/R</td>
<td>Transmit/Receive</td>
</tr>
<tr>
<td>VLSI</td>
<td>Very-large-scale integration</td>
</tr>
<tr>
<td>$A$</td>
<td>Area ($m^2$)</td>
</tr>
<tr>
<td>$C_{OFF}$</td>
<td>Capacitance at OFF state (F)</td>
</tr>
<tr>
<td>$R_{ON}$</td>
<td>ON resistance ($\Omega$)</td>
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<tr>
<td>Symbol</td>
<td>Unit</td>
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<tr>
<td>$E$</td>
<td>Young's (Elastic) Modulus (Pa)</td>
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<tr>
<td>$\dot{E}$</td>
<td>Effective Young’s (Elastic) (Pa)</td>
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<td>Force (N)</td>
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<tr>
<td>$F_e$</td>
<td>Electrostatic force (N)</td>
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<tr>
<td>$F_{ad}$</td>
<td>Adhesion force</td>
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<tr>
<td>dB</td>
<td>Decibels</td>
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<tr>
<td>$P_{out}$</td>
<td>Pull-out force</td>
</tr>
<tr>
<td>$w$</td>
<td>Beam width (m)</td>
</tr>
<tr>
<td>$g$</td>
<td>Gap spacing (m)</td>
</tr>
<tr>
<td>$tg$</td>
<td>Tip gap (m)</td>
</tr>
<tr>
<td>$L$</td>
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</tr>
<tr>
<td>$k$</td>
<td>Spring constant (N/m)</td>
</tr>
<tr>
<td>$k_b$</td>
<td>Beam stiffness (N/m)</td>
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<tr>
<td>$V$</td>
<td>Voltage (V)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Frequency (Hz)</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius of asperity (m)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Surface energy</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>Permittivity of free space (F/m)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard deviation</td>
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<tr>
<td>$\rho$</td>
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<td>velocity on 55V actuation voltage on a set of switches.</td>
<td></td>
</tr>
<tr>
<td>6.18</td>
<td>Adaptive control calculation versus experiment with near-zero landing</td>
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</tr>
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Chapter 1

Introduction

Research is what I’m doing when I don’t know what I’m doing

Wernher Von Braun, German scientist (1912-1977)

Microelectromechanical systems (MEMS) have received great attention in recent years because of their potential for miniaturization, integration and high volume production for various applications in sensors, actuators, wireless communications, micro-robots and biomedical systems. It is created using techniques from the micro-electronics industry, which integrate micro-scale mechanical and electronic functions. The variety of new functionalities exceeding traditional microelectronics opens a new world for micro devices. It is predicted that the impact of MEMS devices will be similar to the impact of current microelectronics on society [1].

MEMS devices can be fabricated using traditional, mature semiconductor manufacturing processes [2]. It is possible to miniaturize with advantageous scaling properties and integrate on wafer with interface circuits for single chip system for multi-purpose applications. The potential to leverage stable manufacturing processes for MEMS devices can result in high volume fabrication to meet market price requirements. However, due to the complexity of the micromechanical parts and the non-standardized packaging procedures reliability issues are an obstacle to widespread application of MEMS devices in today’s marketplace [3]. For the majority of applications, the research and development (R&D) phase is still in progress.
and a key focus is the understanding of reliability issues so as to improve the lifetime of MEMS devices, especially, in harsh environmental applications.

Microelectromechanical devices that are used in radio frequency signal ranges applications are named RF MEMS. For this application RF MEMS switches appear to be the most promising devices. The RF MEMS switch works on the same principle as the simple mechanical moving switch to achieve a short circuit or an open circuit in a transmission line. These types of switches offers superior performance compared to the currently employed solid-state switches. RF MEMS switches exhibit excellent performance in terms of insertion loss, isolation, linearity [3]. Various potential high-end applications can be listed here for RF MEMS switches as: automotive safety, communication systems, test equipment, military and space application. It can be employed to build low-power and low-loss reconfigurable filters, impedance matching networks, digital phase-shifter and switching networks [4].

However, in common with all MEMS devices, the reliability risk of RF MEMS switches is an important factor restricting the transfer of RF MEMS switches from research labs to commercial production lines. MEMS switches reliability refers to their ability to survive to either a large number of operational cycles and withstand long-term stress without significant degradation of both mechanical and electrical performances. In addition, switching high power levels temperature, shock and vibration also affect the life-time of RF MEMS switches [3]. More importantly, most of these physical and environmental phenomena are closely related to each other and have a high interaction when predicting their contribution to switch failure mechanisms. Due to these interactions the reliability aspects of RF MEMS are complex and not completely understood. Therefore, reliability assessment should be concerned in all phases from device conception to design of control strategies to make a successful product. Following these ideas, this work concentrates on development of a reliability control framework for RF MEMS switch.

Figure 1.1 shows the top view of the SEM micrograph of a SPST ruthenium contact RF-MEMS switch used in this work. The prototype is provided by Analog Device with non-disclosed design and fabrication technologies. More details provided throughout the thesis. The minimum actuation voltage is around 45V. Simulation
1.1 Goals of the dissertation

Specifically, the research presented in this thesis focuses on the development of models, designs and control methodologies to reduce/remove some of the effects that might lead to failure mechanisms as mentioned above. Many research and develop-
ment works on reliability issue have been developed and reported in literature. The
goals of this dissertation are to develop the novel simple but comprehensive and
accurate methods to solve effects related to reliability of MEMS ohmic switches.
Following that, the ultimate objective is to improve the lifetime of the RF MEMS
ohmic switches. This research only concentrates on solving some of the most notable
operation issues. Primary goals and specific tasks forming the structure of the work
are:

- To provide a thorough comprehensive analysis of the pull-in instability phe-
nomena of cantilever beam switches. This is is one of the crucial factors
affecting the lifetime of ohmic switches. Whereas the pull-in position is an
important factor. However, the analytical analysis has not been found in any
analytical model from previously published works. Beyond a certain critical
point, the structure under electrostatic force is no longer balanced. The beam
then becomes mechanically unstable and spontaneously collapses to the con-
tact which leading to a high speed interaction. This phenomenon can cause
serious detrimental effects on the reliability of metal-contacts. The generalized
comprehensive pull-in model will be developed for better understanding of the
phenomenon and for predicting the pull-in actuation voltage. The model can
be expanded further on the design objective to avoid instability issue.

- To develop the guidance for an engineer at the initial phase to design a MEMS
cantilever switch without the pull-in instability phenomenon. The analytical
formulation on dimensional design of an electrostatic actuated cantilever beam
without pull-in instability would provide a handy tool for design engineer.
Once the pull-in instability is removed, this in turn will translate into a reduced
contact speed and force, therefore, the contact degradation effect is expected
to be reduced.

- To develop a simplified 2-D simulation model that includes all major parts
of the MEMS ohmic switch: the beam, nonlinear contact interaction, electro-
static load and squeeze-film damping. The complexity of contact roughness
will be heavily studied to develop a multi-asperities model that can capture
well the interaction phenomena. The dynamic of contact resistance and the
bouncing phenomenon can be evaluated based on the simulation model. The developed model is an effective tool to evaluate some of the degradation effects including both cycling effects and long term hold down stress effects. It provides insight into the behaviour of the switch. Therefore, the model and the evaluation method can be used in lifetime evaluation of the switch.

- To develop a closed-form analytical model to define a pre-shaped driving actuation wave-form to reduce the bouncing effect of a MEMS ohmic switch. An open-loop waveform is designed to close the switch with low impact speed. The bouncing issue can be eliminated. The method is then can be adapted to the variance of switches due to fabrication tolerances or drift during operation.

### 1.2 Organisation of the dissertation

This introduction is the first of the seven chapters in the dissertation. The remainder of the thesis is organized as follow.

**Chapter 2** provides a general literature review of MEMS ohmic switches and the state of the art of works in literature related to MEMS switches field with more orientation on ohmic contact switches and their reliability issues. This part does not intend to recite all the knowledge in the field, but to give an overview of research activities and the motivation in the field. Some induced failure mechanisms and design considerations are reviewed in this chapter. An extensive literature reviews for relevant topic is presented inside each chapter.

**Chapter 3** looks at the design of a typical cantilever MEMS ohmic switch. This chapter investigates the pull-in instability of cantilever switches with partial electrode. A generalized model of the pull-in voltage is presented here. The developed models are validated by comparison with commercial Finite Element Model (FEM) simulations results and are also compared with models from previously published works.

**Chapter 4** develops the guidance for no pull-in instability switch design. Follow this guideline, the switch closes to the contact before the instability occurs, therefore
the contact speed and force can be considerably reduced. The FEM validations are carried out to confirm the accuracy of the model.

Chapter 5 describes the development of simplified 2-D model of MEMS ohmic switch. A multi-asperity nonlinear contact with adhesion are considered and integrated into full dynamic switch model. Based on this model, the dynamic performance of the switch is simulated using the Finite Difference Method (FDM). The developed model is used to simulate the deformation of the contact, the switching speed, the tip displacement and the bouncing behavior of the switch. The model assists evaluation of the switch dynamic behaviour including contact bounce and can allow evaluation of the contact physics of material. Contact resistance and contact bouncing behaviour are experimentally validated. It is also used to investigate the creep degradation effect of the beam. It provides insight into the contributions of various mechanical factors to the measured changes in switch performance.

Chapter 6 shows an energy-based control design approach to adaptive pulse shaping in RF MEMS ohmic switches. A closed-form solution for shape a waveform to minimize impact speed of the switch at the contact point is developed. The method can easily adapt to the variance of pull-in voltage by imperfect manufacturing or during operation. The method results are validated simulation experiment for a set of cantilever switches.

Chapter 7 is a concluding summary of the work. It assesses the contribution of the research programme and looks ahead to future directions for reliability improvement.
Chapter 2

RF MEMS switch and its reliability - State of the art

One who develops future intellectual pursuits by understanding the research and works created by notable thinkers of the past

Unknown

Radio frequency (RF) switching devices are now extensively used in network applications. For the next generation of high-end RF switching networks applications, it is essential for new switching technologies to have of low power, low insertion loss, high isolation, high linearity and potential for integration. RF MEMS switches have emerged as an excellent candidate for this requirement. In this chapter, a brief overview history of MEMS switch technologies is presented. This is followed by a review of reliability issues on MEMS switch, with a particular focus on the ohmic contact switch and its reliability.

2.1 RF MEMS switches

MEMS (Micro Electro Mechanical Systems) are micro-scaled integrated devices or systems that combine electrical and mechanical components. The term first ap-
2.2 What does the RF application need

appeared in the middle of 80s, however the technology history began earlier. A gold resonating MOS gate device fabricated in 1967 [5] is generally accepted as a first MEMS device. Its excellent electrical signal characteristics attracted many research institutes and universities to this topic [6]. After a longer than anticipated development cycle the technology matured and was commercialized at the end of the 90s, with the development of a stable fabrication technology. MEMS devices such as sensors and actuators can be co-fabricated with control and readout circuits on the same wafer. Since then research on MEMS has emerged in broader applications such as sensors, communications, biotechnologies, etc.

MEMS switches are MEMS devices which have the same operating principle as the simple mechanical moving switch to achieve a short circuit or an open circuit in a transmission line. It inherits the exceptional RF characteristics of mechanical switches but in the micro-world. It comprises two distinct parts of the actuation (mechanical) section and the electrical section. The movement can be vertically or laterally depending on the layout. There are several methods to actuate the beam such as electromagnetic [7], magneto-static [8], thermal [9], electrostatic [6]. Of these, electrostatic actuation stands out to be the most promising method as it compatible with current integrated circuit (IC) fabrication technology [10]. In this work, only electrostatically actuated MEMS switches are considered.

2.2 What does the RF application need

Radio frequency (RF) switch refers to the devices that are used for switching and manipulation of RF signals ranging from 300 kHz to 300 GHz. The RF characterization of switch is defined by the S-parameters (scattering parameters) of the RF signal passing through them. They describe the efficiency of the operation of the RF switches. Some Figure-of-merit parameters are described as below [11].

- **Insertion loss** is specified when the switch is in the ON state. It is the transmission coefficient $S_{21}$ (dB), between input and output terminals when signal is transmitted though. This parameter quantifies the power loss and
2.2 What does the RF application need

voltage attenuation caused by parasitic capacitance \( C_{ON} \), inductance and resistance \( R_{ON} \) of switch. Capacitance varies with frequency, therefore the parameter is frequency dependent feature. The lower the insertion loss the better the operation of the RF switch.

- **Isolation** is defined as the magnitude of a signal that gets coupled across switch in the open state. Similarly to the \( S_{21} \) parameter as for insertion loss but measured when the switch is in the OFF state. Isolation also depends on frequency. It is mainly due to the parasitic capacitance \( C_{OFF} \) between conducting parts. The good switch should have high isolation factor.

- **Switching speed** represents the transition time for toggling between switch states (ON-OFF). It is the time required for the switch to respond at the output due to the change in control voltage. High switching speed is expected for a better performance of the RF system.

- **Bandwidth** is the frequency limitation that the switch can carry with acceptable signal quality. The losses due to parasitic capacitance and resistance are frequency depended and are the main causes limiting the performance at high frequencies of the switch.

- **Power handling** is the power level that switch can handle without early noticed damages.

- **Actuation voltage** is the minimum voltage should be applied to toggle the switch between states. Low voltage level is required for compatible with the rest of the circuits in the system.

- **Lifetime** refers to their ability to survive either a large number of cycles or a long-term stress without significant degradation of both mechanical and electrical performances.

Among the switch technologies used in RF systems, electromechanical (EM) relays offer the best high frequency performance in terms of high isolation, low insertion loss, and good power handling (up to several watts), but are large, slow
2.3 RF MEMS switch types

RF MEMS switchers allow the propagation of an RF signal from an input to an output in one state and block it in the other state. Stability in one or both of states is achieved by an external energy source.

So far, a variety of switch concepts with many interesting features have been published. In the past decades, the R&D in both industry and academia has mainly focussed on packaging, material, process scaling, device integration and control solutions to improve the reliability and signal quality of RF MEMS switches. Depending on the configuration MEMS switches can be divided into two types: Ohmic switches and capacitive switches according to the nature of contact between the movable membrane and the underlying transmission line.
2.3 RF MEMS switch types

2.3.1 Ohmic switches

An ohmic switch has a metal to metal contact to conduct the switches signal in the closed position. It is also called a metal-contact switch.

Figure 2.1 shows the basic structure of an ohmic RF MEMS switch. It consists of a conductive beam suspended from an anchor over a break in the transmission line. The normal condition is usually OFF. Metal to metal contacts switches are used in series switching. Sufficient potential between the electrode and the beam pulls the beam down-ward to the closed position. Typical Figures of merit are the values of the contact resistance $R_{ON}$ and the OFF state capacitance $C_{OFF}$. In which, $R_{ON}$ is expected to minimize insertion loss, whereas $C_{OFF}$ is expected to be high for high isolation factor.

In the ON-state, the metal contacts are formed as the path of signal propagation. There are some configuration types, for the type drawn in Figure 2.1, the signal enters the anchor, travels across the beam length through the metal contacts to the pad. The RF ranges for ohmic switches is DC to 100GHz. The limit frequency is decided by the parasitic capacitance between the metal contacts.

To date, a variety of RF MEMS switch designs with interesting features have been published. The first MEMS switch specifically designed for RF application was developed by L. K. Larson [13] in 1991. The technology is compatible with conventional GaAs fabrication techniques. The switch was proved to have good RF signal performance. Its insertion loss was less than 0.5dB and its isolation
2.3 RF MEMS switch types

Table 2.1: List of state-of-the-art RF MEMS ohmic switches

<table>
<thead>
<tr>
<th>Published Year</th>
<th>Ref.</th>
<th>Pull-in voltage (V)</th>
<th>Switching speed (μs)</th>
<th>$R_{ON}$ (Ω)</th>
<th>Insertion loss (dB)</th>
<th>Isolation (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>[13]</td>
<td>100</td>
<td>N/A</td>
<td>N/A</td>
<td>0.5</td>
<td>35 @45GHz</td>
</tr>
<tr>
<td>1995</td>
<td>[14]</td>
<td>28</td>
<td>30</td>
<td>0.22</td>
<td>0.12</td>
<td>50 @4GHz</td>
</tr>
<tr>
<td>1997</td>
<td>[15]</td>
<td>30-400</td>
<td>N/A</td>
<td>1-1.5</td>
<td>0.15</td>
<td>40 @4GHz</td>
</tr>
<tr>
<td>1999</td>
<td>[16]</td>
<td>19</td>
<td>300</td>
<td>0.5</td>
<td>0.5</td>
<td>54 @0.9MHz</td>
</tr>
<tr>
<td>2000</td>
<td>[17]</td>
<td>25-60</td>
<td>200</td>
<td>0.3-0.4</td>
<td>0.15</td>
<td>36 @40GHz</td>
</tr>
<tr>
<td>2000</td>
<td>[18]</td>
<td>5</td>
<td>500</td>
<td>10</td>
<td>1</td>
<td>40 @1GHz</td>
</tr>
<tr>
<td>2001</td>
<td>[19]</td>
<td>35</td>
<td>1</td>
<td>1-2</td>
<td>0.1-0.2</td>
<td>N/A</td>
</tr>
<tr>
<td>2001</td>
<td>[20]</td>
<td>30-60</td>
<td>N/A</td>
<td>N/A</td>
<td>0.3</td>
<td>50 @2GHz</td>
</tr>
<tr>
<td>2004</td>
<td>[21]</td>
<td>29</td>
<td>N/A</td>
<td>1.6-4.5</td>
<td>0.8</td>
<td>20 @13GHz</td>
</tr>
<tr>
<td>2007</td>
<td>[22]</td>
<td>68</td>
<td>N/A</td>
<td>N/A</td>
<td>0.4</td>
<td>25 @20GHz</td>
</tr>
<tr>
<td>2008</td>
<td>[23]</td>
<td>90</td>
<td>5</td>
<td>1</td>
<td>0.1</td>
<td>30 @2GHz</td>
</tr>
<tr>
<td>2011</td>
<td>[24]</td>
<td>61</td>
<td>6.4</td>
<td>1-3.8</td>
<td>N/A</td>
<td>21 @6GHz</td>
</tr>
<tr>
<td>2011</td>
<td>[25]</td>
<td>85</td>
<td>30</td>
<td>0.2</td>
<td>0.56</td>
<td>51.4 @12GHz</td>
</tr>
</tbody>
</table>

was greater than 35 dB up to 45 GHz. However, the required actuation voltage was very high at 100V. Since then, many types of switches have been developed with different structures and methods. An incomplete chronological list of RF MEMS ohmic switches can be seen as in Table 2.1.

It is interesting to note that, even the high requirement of actuation voltage is considered to be one of the drawbacks for device integration, reducing the pull-in voltage is not the main purpose of MEMS switches design. No clear trend of pull-in voltage reduction is found for switches cited in Table 2.1 for the last two decades. Full wafer level integration does not seem to be the trend but is now towards a system in a package solution allowing integration of high voltage charge pump die from an optimised DMOS/LDMOS process with a MEMs switch die from an optimised MEMS process.

In 1995, a SiO2 cantilever switch with gold contacts was introduced by Rockwell Science Center [14]. It showed very good characteristics on insertion loss (0.12 dB) and isolation (50 dB) up to 4GHz frequency.

In 1997, Northeastern University and Analog Device Inc. published an electro-
2.3 RF MEMS switch types

Figure 2.2: (a) Electroplated nickel cantilever switch, Analog Device, 1997 [15]. (b) Electroplated gold cantilever switch, Analog Device, 1999

plated nickel cantilever switch, Figure 2.2(a) [15]. Further development of this work demonstrated a gold MEMS switch in 1999 as shown in Figure 2.2(b). Despite a narrow contacts gap, the isolation was quite good (40dB at 4GHz) and a low contact resistance (1-1.5Ohm) resulted in very low insertion loss of 0.15dB.

In 1999, Komura et al. [16] introduced a micro machined relay (MMR) consisting of a glass substrate and a movable element made of single crystal silicon with sputtered gold adopted as contact material. The switch showed some of promising characteristics such as the drive voltage was only 24V, the power consumption was 0.05mW and good RF performance up to a few GHz. However, the switching speed was very low at 300μs.

In 2000, authors from KAIST presented an interesting switch for very low pull-in voltage (~5V) without sacrificing device performance [18]. It utilized torsion springs and leverage techniques to reduce the actuation voltage. The similar push-pull concepts to reduce actuation voltage are also found in various independent works, such as [26].

A system-on-a-package (SOP) integrated solution was presented by [20] in 2001. An electrostatically actuated cantilever switch (~50V actuation voltage) with very good RF performance was integrated with a charge pump electronic chip into one package. The significant progress demonstrated the integration of RF MEMS switches and drive circuits in a portable wireless application.
2.3 RF MEMS switch types

In 2008, Costa et al. [23] published a high performance/high power RF MEMS contact switch based on a 0.18/0.5 RF CMOS thick Silicon-On-Insulator (SOI) technology. The silicon SOI technology also includes the integration of a solid-state RF switch, and a RF power LDMOS transistor with good linear and saturated RF power characteristics in the frequency range between 0.8GHz - 2.4GHz.

In 2011, authors from UCSD [24] presented an electrostatic RF MEMS switch that deploys tethered cantilever topology to achieve low sensitivity to stress gradients. The pull-in voltage was 61V and an actuation time was 6.4\(\mu s\). The design could be used for automated testing equipment and high-performance switching networks.

2.3.2 Capacitive switches

Capacitive switches have a similar actuation scheme as ohmic switches, but a dielectric material layer is designed to isolate two conducting electrodes. This switch type is generally used in a shunt switch configuration. The RF signal is shorted to ground by variable capacitors. The Figures of merit of this capacitance switches is the ratio between ON and OFF state capacitance, \(C_{ON}/C_{OFF}\).

At rest position, the beam is in the up position, the capacitance is of the order of fF (high impedance), which translates as the switch in the OFF state. When sufficient voltage is applied, the induced electrostatic force pulls the beam down. Acting as a shunt capacitor on the coplanar waveguide with capacitance increased to pF levels, which is low impedance for high frequency RF signals, the RF signal is shunted to ground and the switch is in the OFF state. The bandwidth is limited by the capacitance ratio between the ON and OFF states and capacitive MEMS switches are generally applied for RF application ranging from 10-120GHz [3].

Table 2.2 shows characteristics of RF capacitive shunt switches since 1998 till recently. Figure 2.3 shows some SEM micrographs of RF MEMS capacitive switches.
2.4 RF MEMS switches advantages and disadvantages

![Figure 2.3](image)

Figure 2.3: (a) Raytheon, US, capacitive shunt switch [28]. (b) University of Michigan, US, capacitive shunt switch. (c) IMM-CNR, Italia, shunt capacitive switch [36]

<table>
<thead>
<tr>
<th>Published Year</th>
<th>Ref.</th>
<th>Pull-in (V)</th>
<th>( C_{ON}/C_{OFF} )</th>
<th>Insertion loss (dB)</th>
<th>Isolation (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>[27]</td>
<td>14-16</td>
<td>N/A</td>
<td>0.2</td>
<td>30 @40GHz</td>
</tr>
<tr>
<td>1998</td>
<td>[28]</td>
<td>30-50</td>
<td>80-100</td>
<td>0.25</td>
<td>35 @35GHz</td>
</tr>
<tr>
<td>2000</td>
<td>[29]</td>
<td>9</td>
<td>N/A</td>
<td>0.1</td>
<td>50 @1GHz</td>
</tr>
<tr>
<td>2000</td>
<td>[30]</td>
<td>8</td>
<td>600</td>
<td>0.08</td>
<td>45 @25GHz</td>
</tr>
<tr>
<td>2001</td>
<td>[31]</td>
<td>25</td>
<td>N/A</td>
<td>0.3</td>
<td>33 @77GHz</td>
</tr>
<tr>
<td>2002</td>
<td>[32]</td>
<td>N/A</td>
<td>450</td>
<td>0.07</td>
<td>25 @15GHz</td>
</tr>
<tr>
<td>2004</td>
<td>[33]</td>
<td>20</td>
<td>17</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>2008</td>
<td>[34]</td>
<td>30</td>
<td>5</td>
<td>1.6</td>
<td>N/A</td>
</tr>
<tr>
<td>2010</td>
<td>[35]</td>
<td>60</td>
<td>9</td>
<td>0.27</td>
<td>20 @25.7GHz</td>
</tr>
<tr>
<td>2011</td>
<td>[36]</td>
<td>15-20</td>
<td>19</td>
<td>0.8</td>
<td>40 @30GHz</td>
</tr>
</tbody>
</table>

Table 2.2: List of state-of-the-art RF MEMS ohmic switches

2.4 RF MEMS switches advantages and disadvantages

The attraction of MEMS switches is that they offer the performance of EM switches with the size and low cost of SS switches. As the MEMS switch employs a purely mechanical element between state switching, there are a number of advantages that cannot be found with conventional SS, especially on the requirements of modern
2.4 RF MEMS switches advantages and disadvantages

Table 2.3: Comparisons of the performance of Electromechanical relay (EMR), Solid State switches and MEMS switches [3]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>EMR</th>
<th>PIN diode</th>
<th>FET</th>
<th>RF MEMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actuation voltage (V)</td>
<td>3-24</td>
<td>3-5</td>
<td>3-5</td>
<td>5-80</td>
</tr>
<tr>
<td>Current (mA)</td>
<td>15-150</td>
<td>3-20</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Power consumption(^1) (mW)</td>
<td>~400</td>
<td>5-100</td>
<td>0.05-0.1</td>
<td>0.05-0.1</td>
</tr>
<tr>
<td>Switching time</td>
<td>&gt;1ms</td>
<td>1-100ns</td>
<td>1-100ns</td>
<td>1-300µs</td>
</tr>
<tr>
<td>Isolation</td>
<td>Very good</td>
<td>Medium</td>
<td>Low</td>
<td>Very good</td>
</tr>
<tr>
<td>Power handling (W)</td>
<td>10</td>
<td>&lt;10</td>
<td>&lt;10</td>
<td>&lt;1</td>
</tr>
<tr>
<td>Size</td>
<td>Small</td>
<td>Small</td>
<td>Very small</td>
<td>Very large</td>
</tr>
<tr>
<td>Cutoff frequency (THz)</td>
<td>0.005</td>
<td>1-4</td>
<td>0.5-2</td>
<td>20-80</td>
</tr>
</tbody>
</table>

\(^1\)Includes driving circuit

...communication systems. Specifically, their advantages include: (1) high linearity; (2) low insertion loss; (3) low power and compatibility with integrated circuits that enables complete system-on-chip RF front-ends. More details of the characteristics and comparison are shown in Table 2.3.

Typical performances for RF MEMS switches can be described as:

- Extremely low loss: Insertion loss smaller than 1dB and return loss better than 36dB have been demonstrated up to 40GHz [17].
- High isolation: They offer high isolation at RF frequencies, around -50dB to -60dB at 1 GHz and rising to -20 to -30 dB at 20 GHz [3].
- Power consumption: Near zero dissipated for operation (except driving circuitry). It is equivalent to FET switches if the driving circuit is included, which is less than 0.1mW.
- High linearity: IIp3 higher than 60dBm. RF MEMS switches are 20-50dB better than GaAs or CMOS devices on handling identical input powers [37].
- High Q-factor: The quality factor is around 50-400 at 2-100GHz [38].
- Power handling: Quite low for metal-contact switches, 500mW [38]. Better for capacitive switches. The power up to 5-10W have been demonstrated for capacitive [39].
2.4 RF MEMS switches advantages and disadvantages

Table 2.4: Advantages and disadvantages of RF MEMS switch

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low insertion loss</td>
<td>High actuation voltage</td>
</tr>
<tr>
<td>High linearity</td>
<td>Low speed</td>
</tr>
<tr>
<td>High isolation</td>
<td>Low power handling</td>
</tr>
<tr>
<td>Zero power consumption</td>
<td>Price</td>
</tr>
<tr>
<td>Miniaturization</td>
<td>Special packing required</td>
</tr>
<tr>
<td>Large bandwidth</td>
<td>Reliability</td>
</tr>
<tr>
<td>Batch fabrication</td>
<td></td>
</tr>
</tbody>
</table>

- Operating voltage: Need high voltage to actuate the mechanical parts, usually at 5-100V depending on structures.

- Hermetic packing: To protect RF MEMS switches from environmental effects, they all require a hermetic package which tends to increase the cost [40].

- Switching speed: Since MEMS switches involve mechanical movement it is intrinsically slower than electron mobility in traditional SS devices. The switching speed is in the order of a few μs [24].

The typical performances show that there are some limitations specific to RF MEMS switches affecting the usability of this promising device. Table 2.4 shows both the potential performance and commercial benefits and the concerns for RF MEMS switches.

As there are clear advantages of RF MEMS switches, the market potential of the device is significant [3]. Therefore the devices are attracting a lot of attention from both academic and industrial researchers with the goals of improving the characteristics such as: signal quality, switching speed, power handling and life-time performance and reliability.

With the involvement of mechanical movement, MEMS devices are intrinsically slower than tradition SS switches. The switching time is in the order of few μs, whereas it is only tens of ns for FET switches. Furthermore, the requirement for high voltage for switch actuation is another factor that has slowed down the usage of RF MEMS in handheld devices.
Among those disadvantages presented in Table 2.4, the life-time and degradation issues are two of the biggest obstacles that block the usability of the RF MEMS switch in commercial products. In other words, reliability is the main focus that needs to be put on MEMS switch concerns. The difficulty is that all the failure mechanisms are strongly correlated to each other, and involve multiple physical domains [41]: mechanical, electrical, thermal, electro-thermal. The demonstration of high reliability is the key factor for a commercially successful RF MEMS switch.

At this point the author would like to emphasize that the goals of this work are developing methods (analysis and controlling) to reduce the degradation factors so as to improve the life-time of an electrostatically actuated RF MEMS ohmic switch. Therefore hereinafter, the discussions and development mainly focus on MEMS ohmic switches.

2.5 RF MEMS switch applications

The two states switching can be used to connect/disconnect the electrical potential/current of the incoming and the outgoing signal of an antenna or subsystem allowing software controlled RF front-end reconfiguration. Depending on the required frequency, either metal contact or capacitive switches are preferred for certain applications [1].

Metal contact (ohmic) switches are suitable for extremely large bandwidth applications (from DC to a few tens of GHz) with high signal quality. The switching speed and the power handling are less important [42]. Capacitive switches with a capacitance ratio of 20-150:1 are usually used for applications with RF frequency of 2-100GHz with narrow bandwidth [39]. Whereas, switched capacitors with an ON/OFF capacitance ratio of 4-10:1 are ideal tuning devices for RF frequency ranges 0.5-100GHz [34][43].

With its outstanding isolation and insertion loss at ultrahigh frequency, it can replace convention SS witches in cellular phones resulting in much lower power consumption, thus achieving longer battery life. It is also an outstanding candidate
2.6 Reliability issues of RF MEMS switches

For satellite applications and next generation of wireless systems such as multi-mode T/R modules. The switching subsystem such as: Transmit/Receive (T/R) switches [44][25], phase shifters [45][46][47], matching network reconfiguration [48], tunable filter [49][50], etc.

Example of Single Pole n Throw (SPnT) and switching matrices can be found in [51]. It can also be used to realise multi-frequency antennas, where the antenna physical dimensions are changed by controlling a MEMS switching bank [52].

Recently research on MEMs switches has focussed on their potential application in digital computation [53]. Publications have also reported usage for micro-scale mechanical computing systems serving in high radiation areas where tradition SS switches cannot operate effectively. A MEMS based mechanical logic (switch) with a near-zero leakage current, nearly ideal on-off state current ratios, sharp voltage to current relationship slopes and potential for intrinsic immunity to radiation have been fabricated [54][55].

2.6 Reliability issues of RF MEMS switches

For all presented applications, especially for military and space purposes, the switches reliability is a critical factor. Some certain criteria related to isolation, insertion loss, linearity are factors to evaluate the figure of merits of a good RF MEMS switches.

MEMS switches reliability refers to their ability to survive either a large number of cycles or a long-term stress without significant degradation of both mechanical and electrical performances. Furthermore, operational environmental conditions such as temperature, shock, high power signal also contribute to the assessment of RF MEMS switches. The required switching cycles and long term stress are generally in the order of tens of billions cycles for years. There is no defined standard for RF MEMS switches reliability. However, a good RF MEMS switch for next generation wireless systems generally should meet a set of figure of merits such as:

- **Life-cycle**: Number of repetitive ON-OFF switching (in the order of tens of billion) without any sort of failure or with small degradation.
2.6 Reliability issues of RF MEMS switches

- **Long-stress life-time**: In some application, the switch might be actuated and kept in stressed state for a long time (in the number of years). The switch is needed to restore to the released state when required.

- **RF-signal performance**: isolation, insertion, RF power handling must be stable over the required lifetime.

- **Temperature range**: Range of temperature that the switch can operate.

The defined values of these parameters vary depending on the target application and are different on products from different manufacturers. The example of an ohmic switch as shown in Figure 2.2(b), was developed at Radant MEMS, Incorporated [42]. Some of the performance characteristics include lifetime of $10^{10}$ cycles for cold switching and $10^7$ cycles for hot switching with on-resistance variation less than 0.2Ω and current handling capability of 1A. RF characteristics of 0.32dB and 33dB of insertion loss and isolation respectively at 2GHz.

Stable performance for long-term reliability is still an open question for MEMS devices in general and RF MEMS switches in particular. The characteristics and applications prospects of RF MEMS switches for future autonomous wireless communication systems are seen. However, the presence of mechanical parts introduces more reliability issues compared with traditional SS switches. Failure of RF MEMS devices relate to many factors such as processing residue, humidity, packaging, dielectric, RF power, material degradation, creep, fracture. Therefore, there is no defined standard for the evaluation of RF MEMS switches reliability and many issues are yet to be resolved [56].

Typical failure mechanisms of MEMS ohmic switches can be divided into four classes: packaging, electrical, mechanical and environmental phenomena. Some failure phenomena are interrelated due to the physical nature of the problem.

### 2.6.1 Packaging induced failures

RF MEMS switches usually require fully hermetic conditions to avoid external humidity, forces or contaminations. For this reason, packaging, the process responsible
2.6 Reliability issues of RF MEMS switches

for hermeticity, is designed to improve the reliability of MEMS devices. Ironically, the encapsulation process might create some additional failure factors on MEMS switches such as stiction, contamination risks, corrosion [57][58][59][60].

Traditional packaging techniques for integrated circuits have proved to be ineffective for RF MEMS switches. It is recognized that on-wafer hermetic packaging techniques are most suited for RF MEMS switches [3]. The typical process steps are: MEMS switches are first released, packaged on-wafer, and then scribed into individual units. Many problems related to these process steps remain for RF MEMS switches and include:

- High temperature (250-500°C) is required: This is not suitable with the release of thin and long structures (such as the switch beam which is usually designed at 0.5 to 2μm thickness and hundreds μm in length). Temperature may bend the membrane by 1-5μm, making the switch unstable.

- Most bonding techniques outgassing organic materials inside the MEMS cavity due to wetting compounds. This process might create serious detrimental effect on the reliability of both metal-contact and capacitive switches [3].

Reliability issues before and after packaging might differ drastically. It is thought to be one of the main reasons blocking the commercialized RF MEMS switches products into market [10]. The realization process of packaging should minimize the mechanical stresses on the switch die and avoid surface absorption of contaminants and moisture which cause stiction.

2.6.2 Electrical induced failures

Electrical-induced failure modes relate to the propagation of the RF signal through MEMS switches and the localized electric field effects. The major problems related to ohmic switches are the micro-welding and electro-migration that degrade the metal contacts layer, whereas for capacitive switches, the dielectric charging is the main failure cause.
2.6 Reliability issues of RF MEMS switches

- Dielectric charging effect:
  This is the injection and trapping of electrons in the dielectric layers of MEMS switches during operation. Charging can happen both when the conducting signal metal layer touches (in contact) the dielectric due to charging current flow or also from charge separation when high field is applied across the dielectrics without contact [61][62]. Factors such as operating temperature, dielectric material and its quality also affect the speed of charge accumulation [63][64]. This effect leads to a built-in potential voltage that shifts the actuation voltage of the switch. The charging phenomena might be positive or negative. The positive shift causes an increase in the pull-in voltage, whereas, the negative shift, if strong enough, holds the beam in the down-state position when the actuation voltage is removed. Similar with other MOS devices, dielectrics in MEMS switches will break down when the electric field goes beyond a certain value. This is major failure cause of capacitive switches.

- Micro-welding:
  The impact forces between the top and bottom metal contacts of ohmic switches involve a very complex interaction of physical phenomena. As the surfaces of contacts are rough, when the switch is in the contact position, very few micro-spots are in intimate contact between the two surfaces. Therefore, the RF current is concentrated on these spots. As a sequence, the temperature increases, leading to a non-reversible degradation of the materials [65]. Furthermore, it also damage the contacts causing pitting and hardening of the metal layers under repetitive impact. These phenomena affect the equivalent contact resistance, which is strongly related to the insertion loss of RF switches.

2.6.3 Mechanical induced failures

MEMS switches involve repetitive and fast mechanical movements. With time, mechanical degradation mechanisms might appear and impact significantly on the performance and lifetime of switches. The high impact force during switching, which is around 300µN [66] for step input, aggravates other electrically induced failure
2.6 Reliability issues of RF MEMS switches

mechanisms. Creep, fatigue and fracture are some of the most failure modes directly related to mechanical movement.

- **Creep:**

  Creep is the slow permanent deformation of material under mechanical stress over a long period of time [67]. In case of the switch, The beam is deformed permanently (plastic deformation), thus at relaxed position (no potential voltage applied), the value of $C_{OFF}$ is reduced which means the isolation is reduced. This plastic deformation will eventually lead to the stuck-down failure of the switch. Creep is also a temperature-dependent phenomenon. Creep is not to be confused with viscoelastic mechanical behaviour which has been reported for MEMS [68].

- **Fatigue and Fracture:**

  Metals or alloys beams are degraded by material fatigue when subjected to a large number of repetitive mechanical loads. During the switching operation, concentration of stress occur at the anchor (or hinge) areas making these points have a high probability of degradation. The cycling load may lead to the formation of micro-fractures that reduce the stiffness of the beam material. It is widely documented that thin films exhibit much less fatigue than bulk material. The MEMS switch beam is fabricated as a thin film. However, the occurrences of fatigue in several metal thin films have been reported, silver [69], copper [70], aluminum [71] and gold [72].

  Fracture occurs when the stress is greater than the ultimate tensile strength of the material. It is a serious reliability concern. Furthermore, the impacting surfaces have the potential to create debris, fracture components and induce cracks. In relation to the MEMS switch, such failure modes are highly dependent on the approach velocity and contact forces [73]. Asperities on roughness surfaces may break off during impact. The beak-out debris or contaminations will alter the contact properties. This concern combined with hot switching at high RF power result in accelerated contact degradation. Ohmic switches with metal-to-metal contacts are at high risk of these types of failures.
2.6 Reliability issues of RF MEMS switches

Data for fatigue and fracture failure mechanisms are affected by the structure design, materials and fabrication process. Therefore, investigation of these failure modes is device-dependent [74].

2.6.4 Environmental induced failures

The surrounding environmental conditions could contribute a vital role to the failure modes of RF MEMS switches. Temperature, humidity and radiation are the most significant environmental aspects.

- At high temperatures, plastic deformation or material melting can be induced [65], e.g. melting point of Gold is 1060°C. Furthermore, the residual stress of the deposited thin films (the beam) may significantly change due to temperature variation. Cantilever beams can curl positively or negatively due to stress gradients altering the switch performance [75]. The pull-in actuation voltage is observed to be reduced by 0.13 V/°C and the critical buckling stress is reach at 90°C for aluminum alloy membranes [75]. This is especially prevalent during manufacturing and packaging. At critical values of stress, bucking of suspended beams may occur. Material and structure designs are important factors [76] and need extensive studies to optimise the switch tolerance with temperature variation.

- Humidity increases the adhesion forces between contact surfaces. Therefore it affects switch functionality, and might lead to stiction of the beam with the contact pad. Hermetic conditions created by packaging can help to protect RF MEMS switches from humidity factors.

- Small variations of electrical parameters (actuation and release voltages) have been reported on RF MEMS switches under radiation conditions [77]. RF performances have been found to be impaired. This is a particularly critical factor for space applications.
2.7 Operational failure mechanism of MEMS ohmic switches

As discussed in the previous section, both ohmic and capacitive switches share several common failure mechanisms such as fatigue, creep, stiction. However, due to the presence of metal-to-metal contact, ohmic switches have their own failure modes strongly related to the metal contacts, whereas in capacitive switches, the reliability is mainly due to the dielectric charging problem. The main operational failures for RF MEMS ohmic switches are discussed in the following sub-sections.

2.7.1 Contact degradation

The metal contacts are the most crucial part of RF MEMS ohmic switches. They determine the ON state resistance which is directly related to the insertion loss of the RF signal. They also determine the current handling capability which represents the level of RF power that can be switched.

The high impact velocity is one of the main issue affects to the degradation of metal contacts [78][79]. It is highly dependent on the cycling mode [80] and power handling. This failure mode is identified by measuring the change in resistance over time phenomenon. Furthermore, the switch may bounce a few times before making permanent contact. The repeated instantaneous high impact and the high current concentration on a few asperities can cause wear, melting, welding, hardening or pitting of the contact material and distort the crystal structure of the contacts [81][65]. One or more of these mechanisms in combination can cause material to be transferred between the contact surfaces. The situation is exacerbated by the high contact force and hot switching mode operation. As a result, Contact resistance can either increase or decrease, both undesirable effects.

This leads to degraded performance of RF signal, e.g. high insertion loss, therefore the lifetime of the switch is limited due to low quality performance.
2.7 Operational failure mechanism of MEMS ohmic switches

2.7.2 Beam degradation

This mechanism also relates to the cycling effect of the switch. Cracks or fractures commonly appear near the anchor (hinge) areas due to high stress induced to this area. Experimental results in [72][82] were observed this degradation for gold beam switches. The fractured part leads to catastrophic failure of the switch. The fracture on the contact areas might create some debris or contamination on the surfaces.

If the stress concentration near the anchor higher than the ultimate tensile strength, it can result in plastic deformation of the beam. The beam is gradually tilted down to the contact. It reduces the isolation at OFF state of the switch. This effect eventually leads to failure as the closed state is permanent since the restoring force reduces as the beam actuation gap is reduced.

2.7.3 Stiction

This failure mode can be induced in various ways. When the sacrificial layer is removed in the etching process, a meniscus can be formed between substrate and moving parts. In wet etching, during removing liquid, surface tension forces may pull the beam into contact resulting in a stuck-down or stiction failure. Another cause is plastic deformation due to overstress induced on the beam resulting in a permanent tilting of the beam (creep). Furthermore, stiction happens as a combined mechanism of contact and beam degradations. The repeated instantaneous switching might lead to fatigue induced on the beam. The restoring force is reduced. The contact degradation phenomena as mentioned can result in undesirable surface smoothness which in turn causes an increase in the adhesion forces (capillary forces, hydrogen bridging, van der Waals forces) between two metal contacts. If the restoring force of the beam is smaller than adhesion force between contacts, the beam cannot restore back to its original position [83].

All the general observed operation failures are exacerbated by some extreme conditions such as: imperfect fabrication process, high power switching, humidity
effects, temperature changes, etc. The rules and combination mechanisms of failures are very complicated and need more investigation.

2.8 Some considerations for electrostatically actuated RF MEMS ohmic switches

RF MEMS switches operating on electrostatic actuation are the most commonly pursued designs in terms of integration and reliability. The realization of an electrostatic actuation MEMS switches for RF application relates to many aspects. This work has focussed on some selected, typical design problems related to operational aspects.

2.8.1 Design

This section addresses some typical design problems on the device level of electrostatically actuated metal-contact switches. There are several parameters such as actuation voltage, restoring forces, contact resistance, switching speed, gap and reliability that are tightly coupled to each other. A good switch design is therefore a compromise between some of these conflicting goals. Figure 2.4(a) is a side-view schematic of a simple electrostatically actuated cantilever switch, and Figure 2.4(b) shows its equivalent simplified model for cantilever-spring and electrostatic force. The overall goal is to balance these parameters to achieve the desired switch performance, expressed in the actuation voltage, switching speed, on-resistance, isolation and the reliability.

The electrostatic force \( F_e \) and the restoring spring force \( F_s \) are given by \[ F_e = \frac{\varepsilon_0 A V^2}{2(g - y)^2} \quad (2.1) \]

\[ F_s = ky \quad (2.2) \]
Some considerations for electrostatically actuated RF MEMS ohmic switches

Figure 2.4: (a) Schematic side-view of a cantilever based ohmic switch. (b) Simplified spring-mass model

where \( \varepsilon_0 \) is the permittivity of free space, \( g \) is the initial gap, \( V \) is the potential between the electrode and the beam, \( y \) is deflection of the mass, \( A \) is the electrode area, \( k \) is the spring constant reflecting the mechanical stiffness.

The switch is closed if the electrostatic force is larger than the mechanical restoring force. Due to the quadratic dependence of the electrostatic force on the gap height over the linear increase in mechanical restoring force of the beam, the beam is pulled down and the beam-to-electrode gap decreases. The pull-in instability point is the moment at which the attractive electrostatic force exceeds the mechanical restoring force and the stability of the equilibrium is broken. This critical distance is always at \( y = \frac{1}{3}g \) for the simplified model shown in Figure 2.4(b) [3]. This point is called pull-in instability position. Thus, the pull-in voltage of the switch can be estimated by

\[
V_{p1} = \sqrt{\frac{8kg^3}{27\varepsilon_0A}} \tag{2.3}
\]

Equation (2.3) shows that the pull-in voltage can be predicted and estimated in the design phase. Currently, the pull-in voltage of electrostatic switch designs are between 5-100V to achieve sufficient performance. However, the voltage level
2.8 Some considerations for electrostatically actuated RF MEMS ohmic switches

Table 2.5: Some design considerations of RF MEMS ohmic switch

<table>
<thead>
<tr>
<th>Items</th>
<th>Parameter</th>
<th>Desired</th>
<th>How</th>
<th>For</th>
<th>Against</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Actuation voltage</td>
<td>Low</td>
<td>↓(2.5),↑(3)</td>
<td>Compatibility</td>
<td>2,3,5,7*</td>
</tr>
<tr>
<td>2</td>
<td>Restoring force</td>
<td>High</td>
<td>↑ thickness</td>
<td>Avoid stiction</td>
<td>1,7</td>
</tr>
<tr>
<td>3</td>
<td>Size</td>
<td>Low</td>
<td>↑(1), ↓(2)</td>
<td>Integration</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Contact force</td>
<td>High</td>
<td>↑(1), ↓(2)</td>
<td>8</td>
<td>1,2</td>
</tr>
<tr>
<td>5</td>
<td>Gap</td>
<td>High</td>
<td>6</td>
<td></td>
<td>1,7</td>
</tr>
<tr>
<td>6</td>
<td>Isolation</td>
<td>High</td>
<td>↑(5)</td>
<td>Performance</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>Switching speed</td>
<td>High</td>
<td>↑(1), ↓(2,5)</td>
<td>Performance</td>
<td>1,2,5,9</td>
</tr>
<tr>
<td>8</td>
<td>Contact resistance</td>
<td>Low</td>
<td>↑(4)</td>
<td>Performance</td>
<td>1,9</td>
</tr>
<tr>
<td>9</td>
<td>Contact reliability</td>
<td>High</td>
<td>↓(4,7)*</td>
<td>Performance</td>
<td>4,7,8</td>
</tr>
</tbody>
</table>

*Items number in the first column

Currently applied in electronic devices is less than 5V. Therefore, additional driving circuits (amplifiers) are generally required for RF MEMS switch actuation. Ideally, \( V_{pi} \) is required to be as small as possible to reduce the complexity of these driving circuits. According to (2.3), the pull-in voltage can be controlled by reducing the cantilever-spring constant, decreasing the gap or increasing the electrode area. However, the contradiction appears in this requirement. Cantilever-spring, \( k \), also drives a restoring force of the switch. To prevent stiction problem, a high value of \( k \) is desired. The initial gap, \( g_i \), should not be very small to avoid fabrication errors and to achieve RF isolation specifications. Increasing electrode area means increasing the size of the switch and also decreases RF isolation. Furthermore, high contact force \( (F_c) \) is desired to reduce the contact resistance (reduce insertion loss for RF signal). This requirement calls for high actuation forces, which is contrary to the previous target. In addition, as the contacts degrade at faster speed if they are intensively stressed, high force value affects the contact reliability. Table 2.5 shows some design considerations of a typical RF MEMS ohmic switch.

Based on the previous analysis, the most important switch performance parameters are listed in Table 2.5 with their desired objectives and methods of achieving those objectives. The last two columns in Table 2.5 show the design motivation of each parameter objective and highlights the contradiction between those realizing objectives. For example, the objective is to have a high switching speed to improve the performance of the system. High actuation voltage, '1', low restoring force, '2',...
Some considerations for electrostatically actuated RF MEMS ohmic switches

and small gap, '5', are expected to contribute to realizing this objective. However, modifying these parameters to optimize switching speed is contrary to the desired objective as specified for them. Furthermore, high speed switching creates high impact force, which is one of the main reasons leading to contacts failures. Therefore, it conflicts with the contact reliability objective. Thus, to have efficient RF MEMS ohmic switch design, the compromised parameters should be carefully calculated. It is different from design to design and is based on the requirements of the overall systems.

Items 8 and 9 are not only contradicted in contact forces and switching speed but also in material metal used. Low resistance, high conductivity metal (i.e. gold) is usually soft therefore its reliability is sensitive to the contact force level applied.

Table 2.5 reflects that it is not easy to create an effective switch design with low actuation voltage (for system compatibility), reasonable restoring force (for stiction avoidance), small size (for integration and cost), high initial gap (for high isolation), high contact force (for low resistance). The RF MEMS switches designers are aware of these problems and many concepts have been proposed:

- **Curled Cantilever/membrane:**
  This is the most popular design and can be found in many works [15][19]. Figure 2.2 shows an example of cantilever design.
  
  **Advantages:** low actuation voltage; large contact distance.
  
  **Disadvantages:** difficult to control residual stress.

- **Push-pull:**
  The design includes two separate push and pull actuations. The additional electrode can be designed on top [27], the other sides of the rotational axes [18], or separate contact [84].
  
  **Advantages:** active closing, opening; low actuation voltage.
  
  **Disadvantages:** fabrication complexity.
2.8 Some considerations for electrostatically actuated RF MEMS ohmic switches

Figure 2.5: Schematic view of the curved electrode actuator [85]

- **Curved or stepped electrodes:**
  
  This structure, Figure 2.5, is typically used in lateral actuators because of its simple fabrication [85].
  
  **Advantages:** suitable for lateral actuators, increase contact distance.
  
  **Disadvantages:** not applicable for vertical actuators. However, it is not a problem for switch design.

- **Free-moving membrane actuator:**
  
  The switch is designed with completely unsupported membrane moving between electrodes [29]. Therefore, theoretically there is no restoring spring energy.
  
  **Advantages:** very low actuation voltage.
  
  **Disadvantages:** complex fabrication, friction and stiction problems,

2.8.2 Pull-in instability and closing speed

For simplified spring-mass model as shown in Figure 2.4(b), the instability point, where the stability of the equilibrium is broken, is found at one-third of the gap regardless of design geometries. However, the idealized spring-mass model does not
2.8 Some considerations for electrostatically actuated RF MEMS ohmic switches

account for nonlinearity aspects of the geometry, therefore the actual instability point is not always at 1/3 of the gap. Petersen [12] employed a square-law beam shape to obtain a pull-in position close to 0.5 (half of the gap) under a fully distributed electrostatic force. The value of 0.45 to 0.47 of the normalized pull-in position was confirmed by Finite Element Method (FEM) simulation in [86] for the full electrode case.

Wherever the pull-in instability is present, this phenomenon creates a high impact force at closing. It is highly detrimental to the performance and lifetime reliability of the devices. The leveraged bending effect can be used to achieve full gap travel without reaching the pull-in instability point [87]. This method is not only useful for switches to reduce contact force, but also useful for extending the travel range of analog-tuned MEMS devices. No general analytical model to accurately calculate the pull-in instability point of partial electrode designs was found in literature. A suitable model was developed in this work and was applied to switch design to actuate without occurrence of pull-in instability.

Switches designed for actuation with the pull-in instability have higher closing speed and contact forces than the no pull-in instability design. However, for many fabricated devices without pull-in instability, the closing speed is still high and has created several problems [88][89]. Due to high impact forces on closure, the switch bounces backward several times, interrupting the output signal and increasing the device closing time. The high impact forces and repeated closures are believed to be the main factors in contact degradation. To achieve the required performance for commercial applications, control techniques can be used to improve the MEMS device actuation characteristic.

Many works on both open-loop and closed-loop control approaches have been reported. In terms of cost and complexity with limited space and power consumption in industrial applications, the open-loop (pre-shaping) method is preferable. Open-loop methods based on tailoring the waveform actuation mainly employ the dual-pulse technique [90][66]. The method comprises two pulses. The actuation pulse has a high voltage but short duration which provides the necessary amount of energy to actuate the structure to the contact with near-zero velocity. The hold voltage is then
2.8 Some considerations for electrostatically actuated RF MEMS ohmic switches

Table 2.6: Some potential contact metals, alloys for RF MEMS switches [91]

<table>
<thead>
<tr>
<th>Metal</th>
<th>Symbol</th>
<th>Resistivity ($\mu\Omega cm$)</th>
<th>Hardness (MPa)</th>
<th>Melting point ($^\circ$C)</th>
<th>Process complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>Au</td>
<td>2.2</td>
<td>250</td>
<td>1060</td>
<td>simple etch</td>
</tr>
<tr>
<td>Gold Nickel</td>
<td>AuNi5</td>
<td>12</td>
<td>1600</td>
<td>1040</td>
<td>simple etch</td>
</tr>
<tr>
<td>Rhodium</td>
<td>Rh</td>
<td>4.3</td>
<td>2500</td>
<td>1960</td>
<td>difficult etch</td>
</tr>
<tr>
<td>Ruthenium</td>
<td>Ru</td>
<td>7.1</td>
<td>2700</td>
<td>2330</td>
<td>difficult etch</td>
</tr>
<tr>
<td>Iridium</td>
<td>Ir</td>
<td>4.7</td>
<td>2700</td>
<td>2460</td>
<td>difficult etch</td>
</tr>
<tr>
<td>Tungsten</td>
<td>W</td>
<td>5.5</td>
<td>&gt;3000</td>
<td>3420</td>
<td>simple etch</td>
</tr>
</tbody>
</table>

applied upon contact closure to keep the switch from opening. No bouncing issue and near-zero closing speed have been demonstrated. However, current approaches on tailoring the waveform are non-explicit (numerical or semi-numerical) methods, which are difficult to apply from an engineering perspective. An analytical closed-form solution for shaping a waveform to minimize impact speed of the switch is desirable for practical fields. The method developed can easily adapt to the variance of pull-in voltage by imperfect manufacturing or during operation.

2.8.3 Contact material choice

As discussed in the previous subsections, the contact requirements are high conductivity, stable and high reliability. The selection of contact material depends on the hardness, resistivity, yielding point, melting point and even process difficulty. Soft metal like Gold (Au) has good conductivity even under low stress, but the yielding and melting points are low. The clean pure Au contact was reported with several issues such as plastic deformation, pitting and hardening damage under repetitive impact [3] to make it not very reliable for MEMS switches. Several other metals were extensively researched on evaluating the desirable trade-offs. Table 2.6 shows several potential metals and alloys could be used for RF-MEMS switches [91].

Figure 2.6 shows the experiment results presented in [91] of contact resistance versus force of several metals. It is observed that AuNi alloy is a good candidate that can compromise adequate conductivity, stability while maintaining high hardness and simple process complexity.
An emerging approach using carbon nanotube (CNT) and gold coating have been tested for MEMS contact by McBride et al. [92]. Experiment results show that the electrical properties are comparable to Au-Au contact whereas the reliability is improved manifold.

### 2.9 Summary

In this chapter, the fundamental of RF MEMS switches have been studied. A brief historical review and some comparison between switch types have been described. The state of the art regarding RF MEMS switches reliability has been presented. The review was focused more on MEMS ohmic switches as this type of switches will be the main object for the later works of this dissertation. From this review, the contact and beam degradation effects appear to be most vulnerable issues related to the operational reliability of MEMS ohmic switches. Besides, the design considerations for an effective switch have also been discussed. Finally, the analytical models on pull-in instability and control methods to reduce the closing speed...
have been reviewed. Extensive reviews of pull-in instability, simulation models and control methodologies will be provided throughout this dissertation within relevant sections.
Chapter 3

Generalized closed-form models for pull-in analysis of micro cantilever beams subjected to partial electrostatic load

*Physics is mathematical not because we know so much about the physical world, but because we know so little; it is only its mathematical properties that we can discover*

*Bertrand Russell, British scientist, (1872-1970)*

This chapter presents a new approach for the pull-in analysis of micro cantilever beams subjected to partial electrostatic actuation. A novel generalized closed-form model for the pull-in instability position and pull-in voltage of a cantilever beam with partial overlap of the beam and fixed actuation electrode is developed. The model includes the effects of non-uniform loading and fringing electric fields. The model is validated with 3D Finite Element Method (FEM) simulations and against other existing empirical and analytical models and is demonstrated to be of equal accuracy with a more generalized model framework.
3.1 Introduction

As discussed in the Chapter 2, actuation voltage (or pull-in voltage) is one of the most important factors in the design of a MEMS switch. In MEMS electrostatic actuator design the well-known pull-in phenomenon must be taken into account when choosing structural materials, device geometry and actuation schemes [45]. Pull-in occurs as a result of a higher rate of increase of electrostatic forces than the mechanical spring force as the gap between the fixed and movable electrodes decreases. The instability point is the position at which the attractive electrostatic force is dominant over the mechanical restoring force, the stability of the equilibrium is broken. It is fundamental to the operation of electrostatic MEMS devices as it reduces the stable range of motion in some tuning applications and it creates high impact forces for contact switching applications.

Modeling and simulation of electrostatic MEMS devices plays an important role in the early design phase in predicting the device characteristics including the pull-in instability. A number of options are available for device simulation. Using finite element method (FEM) analysis with tools such as CoventorWare or ANSYS provides high accuracy results but it is very time consuming and requires the user to learn how to interact with the software tools. Furthermore, even the results of FEM analysis are very accurate, they have implicit physical meaning and therefore do not easily enhance the designers understanding of the role of different design parameters on the design goal. Therefore, analytical expressions are needed for a better insight into the physical characteristics of general devices. Pull-in analysis has been extensively studied in the literature for cantilever beams.

Nathanson et al. [5] simplified a resonant beam as a lumped model and derived the normalized pull-in point at one-third of the gap regardless of the beam structure and the gap distance. This closed-form expression of the pull-in voltage based on the 1/3 (one-third of the gap) is widely used for tutorials introductions to the pull-in instability in MEMS [3].

However, as the general MEMS structure is highly non-linear, the situation is more complicated than a simple lumped model. Petersen [12] employed a square-law
beam shape to obtain a normalized pull-in position close to 0.5 (half of the gap) under a fully distributed electrostatic force. The pull-in expression for a cantilever beam with electrostatic forces applied along the entire length was validated with experiment data. However, the measured normalized pull-in position result was around 1/3 of the gap.

An empirical expression presented by Osterberg et al. [93] was developed by applying data fitting techniques to a large number of data sets. The accuracy is within 1.5% of FEM results under certain conditions. However, the closed-form is based on the crude 1/3 position in accordance with the fitting parameters. The real pull-in position was not analysed and the partial electrode was not considered.

Pamidighantam et al. [94] presented a model that includes the effects of partial electrode configuration into the expression to determine the pull-in voltage for cantilever beams. The obtained results have been compared with ConventionalWare software with deviation of 3.4% to 11.1% on some cantilever configuration cases. However, the effective stiffness was approximated based on uniform load along the beam length although it is in fact non-uniform for electrostatic forces. The pull-in position was not analysed but assumed to be 0.45 for all cases of partial lengths which probably causes some additional error.

A capacitance-based analysis [86] was carried out to analyse the static behaviour of rigid and deformable electrostatic actuators. Normalized pull-in position and voltage was analysed with partial electrode on rigid torsion actuators. However, the similar analysis on deformable actuators with partial electrode was not considered. Normalized pull-in position of cantilever beams with full length electrodes was modeled to be in the range of 0.454 to 0.472 which agreed well with FEM simulation. Furthermore, the pull-in voltage expression for the deformable cantilever was not explicitly determined due to the presence of the second derivative of a complex capacitance expression.

Chowdhury et al. [95] developed a simple efficient closed-form model which achieves high accuracy even for the case of extreme fringing field effect (narrow beams). The approach includes a factor of 0.93 to compensate the linearisation analysis and crude 1/3 pull-in position. However, this compensation factor might
only correct for the fully distributed electrostatic force as it is studied in this case as the effect of partial electrode was not considered.

Recently, several works have been reported to extend the pull-in voltage calculation on a wide range of electrostatically actuated cantilever beams, such as micro curled beams [96], cantilever beams under residual stress gradients [97] or tapered cantilever beams [98]. To the best of authors' knowledge, there is still a lack of work on the pull-in position and voltage analysis of partial electrode cantilever beams which have been found in many devices, especially MEMS switches [99][66].

In this chapter, a correlation between the pull-in position and the partially distributed electrostatic force over the length of a cantilever beam is determined. The expression is derived by numerical calculation of the load-deflection characteristics of the cantilever beam under non-uniform electrostatic force. It is shown that if the fringing field effect is neglected the pull-in position is independent of effective stiffness, as for the lumped model case which is proven in [96]. The drawback of using a uniform load for estimating effective stiffness is addressed. A closed-form expression for the pull-in voltage of a partially distributed electrostatically actuated cantilever beam is derived based on a high accuracy approximation for implicit integral. The model takes into account non-uniform electrostatic force, effects of partial electrodes and fringing field effects. The developed pull-in analysis will provide valuable information and efficient support for prototyping design.

The remainder of the chapter is organized as follows: Section 3.2 describes the theoretical-based on the formulation of pull-in phenomenon, Section 3.3 presents a numerical analysis on evaluating the developed formulation. A generalized closed-form model is established in Section 3.4. Finally, model validations and discussions are presented in Section 3.5.
3.2 Theoretical study

3.2.1 Effective stiffness

Cantilever beams with partial electrodes have been found in many devices [3], and of particular relevance to this work in MEMS switches [89][66]. Accurate modelling of the beam deflection and electrostatic force between the beam and electrode is the basis for accurate prediction of the pull-in behaviour of the devices and this is addressed in this section.

As discussed in the previous section, the approach adopted in the majority of publications has employed a linear effective stiffness when estimating the pull-in voltage. The effective stiffness is considered constant over the whole deflection of the beam. The effective stiffness was generally derived by one of two methods: using an analytical formula for uniform load [93][94], or using FEM simulation for small deflections [86]. For both cases, the effective stiffness is considered to be independent of the deflection of the beam whereas in practice a certain amount of stiffening occurs and can be a significant factor in beam behaviour. In the former method, the load is considered uniform, \(q_0\), along a partial electrode length \(l_e\) as shown in Figure 3.1. The partial load is starting from the tip.

The effective stiffness for partially distributed uniform load seen at the free-end of the beam is given by [94][100]

\[
K_{\text{eff}}^{\text{linear}} = \frac{\text{force}}{\text{deflection}} = \frac{q_0 b}{y(L)} = \frac{2 \hat{E} w t^3}{3 L^3} \left[ \frac{3}{8 - 6\alpha + \alpha^3} \right]
\]

(3.1)

where \(w, t\) are width and thickness of the beam, \(\alpha\) is ratio of electrode length \((l_e)\) over beam length \((L)\). The effective Young’s modulus \(\hat{E}\) is equal to \(E/(1 - \nu^2)\), \(\nu\) is Poisson’s ratio, for wide beams \((w \geq 5t)\), and equal to \(E\) for narrow beams.

For this estimation, at a given value of alpha \((\alpha)\), the effective stiffness is considered constant at any deflection. However, at deflection \(y(L)\), the electrostatic force is non-uniform along the distributed area. This fact will lead to some level of error in the final approximation of the pull-in voltage which uses equation (3.1).
3.2 Theoretical study

The error becomes higher as the gap between the electrode and the beam increases. Hence, including the effect of non-uniform electrostatic force into the calculation will improve the accuracy of the analytical method. The schematic of non-uniform distributed loads on a cantilever beam is illustrated in Figure 3.2. The start point is from the beam tip.

At any position $x$ along the beam and over the electrode, the concentrated load has a magnitude of

$$q_{\text{elec}}(x) = \frac{\varepsilon_0 w V^2}{2 (g - y(x))^2} \, dx \quad (3.2)$$

where $\varepsilon_0$ is the permittivity of free space, $g$ is the initial gap, $V$ is the potential between the electrode and the beam and $y(x)$ is the displacement at location $x$.

The deflection at the free-end due to this concentrated load (3.2) is found as [100]

$$dy(L) = \frac{q_{\text{elec}}(x)x^2(3L - x)}{6EI} \quad (3.3)$$

where $I$ is the second moment of inertia ($I = wt^3/12$)
By integrating over the region of the load, the total deflection is

\[ y(L) = \int dy(L) = \frac{\varepsilon_0 w V^2}{12EI} \int_{L-\alpha L}^{L} \frac{x^2(3L - x)}{(g - y(x))^2} \, dx \]  \hspace{1cm} (3.4)

Similarly, integrating equation (3.2) over the electrode region, the total force is found as

\[ F_e = \int_{l_c} q_{\text{elec}}(x) \, dx = \frac{\varepsilon_0 w V^2}{2} \int_{L-\alpha L}^{L} \frac{1}{(g - y(x))^3} \, dx \]  \hspace{1cm} (3.5)

The effective stiffness, defined as the total force over the total deflection, at
3.2 Theoretical study

deflection $y(L)$ for non-linear stiffness analysis is derived as

$$K_{\text{eff}}^{\text{non-linear}} = \frac{\text{force}}{\text{deflection}} = \frac{F_e}{y(L)} = 6EI \frac{\int_{L-aL}^{L} \frac{1}{(g-y(x))^2} \, dx}{\int_{L-aL}^{L} \frac{x^2(3L-x)}{(g-y(x))^2} \, dx} \quad (3.6)$$

where $y(x)$ is the deflected shape of the beam (also referred to as a shape function of the beam by some authors [94]). It is seen in equation (3.6) that the effective stiffness is non-linear function of the deflection of the beam.

3.2.2 Pull-in instability

Equating the electrostatic attractive force with the restoring force due to beam stiffness, the stationary equilibrium equations of the system can be described as: $F_e - K_{\text{eff}}y(L) = 0$. Solving this equation, a static equilibrium relationship between applied voltage and displacement $y(L)$ can be obtained for both linear and non-linear effective stiffness as

- **Linear effective stiffness**:

  $$V = \sqrt{\frac{\dot{E}Iy(L)}{3\varepsilon_0 w L^3(8 - 6\alpha + \alpha^3) \int_{L-aL}^{L} \frac{1}{(g-y(x))^2} \, dx}} \quad (3.7)$$

- **Non-linear effective stiffness**:

  $$V = \sqrt{\frac{12\dot{E}Iy(L)}{\varepsilon_0 w \int_{L-aL}^{L} \frac{x^2(3L-x)}{(g-y(x))^2} \, dx}} \quad (3.8)$$

The shape function $y(x)$ of the beam can be approximated by a number of functions which have been widely used in literature, such as:
3.3 Numerical analysis

Square-law curvature [12]:

\[ y(x) = \left( \frac{x}{L} \right)^2 y(L) \]  \hspace{1cm} (3.9a)

Cosine trial function [94]:

\[ y(x) = \left( 1 - \cos \frac{\pi x}{2L} \right) y(L) \]  \hspace{1cm} (3.9b)

FEM extracted function [86]:

\[ y(x) = \left( 2.56 - \frac{16.127}{4(\frac{x}{L} + 0.00185)^2 + 6.2786} \right) y(L) \]  \hspace{1cm} (3.9c)

The order for full electrode is arranged from low to high accuracy.

Theoretically, taking the derivative of equations (3.7) and (3.8) with respect to the beam deflection and setting it to zero \( \left( \frac{dV}{dy(L)} = 0 \right) \), the instability position can be calculated. Substituting this value back into equations (3.7) and (3.8), the pull-in voltage can be found [5][3].

However, the derivatives are complicated and an explicit solution cannot be found for the shape functions \( y(x) \) listed in equations (3.9).

As the integrals inside equations (3.7) and (3.8) are complicated and cannot be computed analytically for the shape functions listed in equations (3.9), a numerical analysis is used to find the voltage and deflection relationship, as described below.

### 3.3 Numerical analysis

Numerical integration is used to compute an approximate solution of the definite integrals in equations (3.7) and (3.8). A preliminary CoventorWare [101] simulation was carried out to evaluate some shape functions presented in equations (3.9). Evaluation of the effect of mesh size on the result accuracy showed that with tetrahedral mesh lower than 5\( \mu \)m, the accuracy does not improve while increase the simulation...
3.3 Numerical analysis

Figure 3.3: Beam shapes comparison between CoventorWare simulations and 3 shape functions in equations (3.9). (a) $\alpha=1$. (b) $\alpha=0.1$

significantly. Therefore, 5$\mu$m tetrahedral mesh is used in this work. Figure 3.3 shows two examples of $\alpha=1$ and $\alpha=0.1$. Parameters of the CoventorWare model are shown on the figure. With the deflection at the beam tip ($y(L)$) found from CoventorWare, the deflection along the beam from shape functions in (3.9) are calculated and plotted to compare with beam deflection from FEM simulation results. It is seen in Figure 3.3 that at $\alpha=1$, the beam deflection from Coventor is very close to shape function in (3.9c). Whereas, it is closer to shape function in (3.9b) for $\alpha=0.1$

The absolute errors between CoventorWare simulation and the shape functions of (3.9b) and (3.9c) are shown in Figure 3.4. It is shown that with a small length electrode ($\alpha=0.1$), the error between calculated shape function and simulation is higher for (3.9c), but error from (3.9b) is higher at ($\alpha=1$).

Figure 3.5 shows the average errors of 2 shape functions versus simulation results at different values of $\alpha$. It is seen that at the high value of $\alpha$ ($\alpha >0.23$), error of shape function in (3.9c) is smaller than (3.9b). To reduce the actuation voltage, the electrode length should be designed large ($\alpha$ closer to 1). Therefore, a FEM extracted shape function for a full length electrode (3.9c) is used in this calculation. If the structure is designed with smaller electrode ($\alpha <0.23$) the shape function shown in (3.9b) should be used for higher accuracy.
3.3 Numerical analysis

Figure 3.4: Absolute error between CoventorWare simulations and two shape functions in equations (3.9). (a) $\alpha = 0.1$. (b) $\alpha = 1$

Figure 3.5: Average error between CoventorWare simulations and two shape functions in equations (3.9) versus $\alpha$.

To demonstrate that the model is general, a range of Young’s modulus values from 57 to 80GPa, common with the materials used in MEMS, is considered in the next analysis. To determine the normalized pull-in position, the tip displacement is swept from $q$ to 0 and the voltage corresponding to that displacement is calculated. The point at which voltage is not increasing anymore and the solution enters a physically unrealistic region (dashed lines in Figure 3.6) is the pull-in instability point. This point corresponds to the maximum value of the voltage over the tip-gap range.
3.3 Numerical analysis

Figure 3.6: Graph of stable equilibrium voltage versus deflection. Comparison between linear and non-linear effective stiffness analysis for full load cantilever beams. Properties of the system are shown on the graph.

Figure 3.6 shows an example of plotting the tip deflection versus applied voltage at \( \alpha = 1 \) for both equations (3.7) and (3.8) by the described numerical approximation.

It is seen in Figure 3.6 that the pull-in position \( (y_{pi}) \) and voltage \( (V_{pi}) \) for linear effective stiffness are higher than the counterparts for non-linear effective stiffness at \( \alpha = 1 \). For this full length electrode, the normalized pull-in position is found as 0.442 with non-linear effective stiffness, whereas, it is 0.535 with linear effective stiffness. The pull-in voltages are found as 4.96V and 5.78V for non-linear and linear stiffness analysis respectively.

A comparison of the normalized pull-in position and the pull-in voltage using equations (3.7) and (3.8) as a functions of electrode length are shown in Figure 3.7 for two different MEMS cantilever designs.

Figure 3.7 shows that the discrepancy between equation (3.7) with linear \( K_{eff} \) and equation (3.8) with non-linear \( K_{eff} \) is biggest at full load (\( \alpha = 1 \)). As \( \alpha \) approaches
3.3 Numerical analysis

Figure 3.7: Comparison between linear and non-linear effective stiffness analysis for different electrode lengths of cantilever beams. (a) $E=57\text{GPa}$, $\nu=0.33$, $g=1\mu\text{m}$, $w=50\mu\text{m}$, $L=100\mu\text{m}$, $t=6\mu\text{m}$. (b) $E=80\text{GPa}$, $\nu=0.06$, $g=2.5\mu\text{m}$, $w=10\mu\text{m}$, $L=200\mu\text{m}$, $t=1\mu\text{m}$.

zero the solution converges to the point load solution, as expected.
3.3 Numerical analysis

Figure 3.8: Example of 3-D CoventorWare simulation of cantilever beam with half electrode positioned at the tip end. Beam deflection is presented by color.

Interesting to note from Figure 3.7 is that the normalized pull-in position curve does not change for different properties of the system. The normalized pull-in positions vary around 0.333 (at $\alpha \sim 0$) to 0.442 for equation (3.8) or 0.535 for equation (3.7) (at $\alpha=1$) regardless of the beam properties. This result agrees with the explicit solution for the lumped system as described in [3][5] that the pull-in position is independent of spring stiffness.

Apart from that, as seen on Figure 3.7, in both cases, the pull-in voltage drops significantly as $\alpha$ increases from 0 to 0.5. However, from $\alpha=0.5$ to $\alpha=1$, the pull-in voltage slightly increases for linear stiffness analysis, while it is still dropping, gradually, with non-linear stiffness analysis. The results from linear stiffness analysis for values of $\alpha$ between 0.6 and $\alpha=1$ do not make sense as the longer the electrode (or the bigger the electrostatic force) results in higher the pull-in voltage.

Figure 3.8 shows an example of a 3-D view of a cantilever structure with a drive electrode half the cantilever length by CoSolve FEA in CoventorWare 2010. The electrostatic forces actuate the beam moving downward. The deflection is shown by color bar.
3.3 Numerical analysis

Figure 3.9: Comparison between analysis based on proposed models and FEM simulation for different electrode lengths of cantilever beams, $E=57$ GPa, $\nu=0.33$, $g=2\mu$m, $w=50\mu$m, $L=200\mu$m, $t=3\mu$m. (a) Pull-in position. (b) Pull-in voltage.

Figure 3.9 shows the pull-in position and pull-in voltages calculated by the FEM analysis and numerical calculation using equations (3.7) and (3.8). The pull-in
voltages and positions are derived by performing CoSolve FEA for different electrode lengths. The FEM pull-in position data is not smooth which is believed to be due to the meshing and input voltage resolution of the CoSolve FEA. However, the FEM pull-in position follows the trend of the non-linear stiffness analysis.

It is observed that the largest error is found in the analysis using linear effective stiffness method as approaches unity. The drawback of linear effective stiffness approximation is seen clearly in Figure 3.9. The assumption of a linear effective stiffness leads to a physically incorrect perception that, the pull-in voltage at $\alpha=0.5$ is lower than the pull-in voltage at $\alpha=1$. This method also leads to a high error in the prediction of the pull-in position, especially for the full length electrode case.

The calculation using a non-linear effective stiffness agrees well with the FEM data for both position and voltage analysis. As it improves the model accuracy, a non-linear effective stiffness will be assumed in the remainder of this work.

A small mismatch found between this analysis and the FEM results is believed to be due to the fringing field effect [93] which was not included in equation (3.8). Fringing effect is the magnetic characteristic caused by the shape around directly opposing the magnetic surfaces. With the presence of fringing field effect, higher electrostatic force on the beam is generated at certain voltage, therefore the pull-in voltage is reduced. The pull-in position is also changed accordingly. With the fringing field effect neglected, the curve of normalized pull-in position $y_{pi}/g$ versus $\alpha$ is fixed regardless of beam properties. The fitted curve for non-linear $K_{eff}$ analysis is:

$$\frac{y_{pi}}{g} = 0.3333 + 0.2125\alpha + 0.0623\alpha^2 - 0.3315\alpha^3 + 0.1672\alpha^4$$ (3.10)

As equation (3.10) is derived using the FEM extracted shape for a full length electrode (3.9c) some error is expected due to the slight changes in the real shape of cantilever beams with different partial electrode lengths ($\alpha$). Furthermore, the fringing field effect also affects the pull-in position [86]. However, equation (3.10) would provide a reasonably good approximation of the pull-in position for a device with partial electrode overlap. It is seen that, at a very short electrode length $\alpha \to$
Figure 3.10: Effect of fringing field on pull-in instability position for cantilever beam

0, the pull-in position approaches 1/3 of gap which is similar to the parallel plate capacitor model [5].

Fringing field effect:

For large $w/g$ ratio, the fringing fields can be neglected. However, when the $w/g$ ratio is below 3 ($w/g < 3$) the fringing field component should be considered. An empirical formula for the capacitance between VLSI on-chip interconnects as employed in [95] would accurately quantify the fringing field effect between the electrode and the beam. The expression is too complicated to apply in this analysis and a simplified approximation by changing the effective beam width is employed in this work.

The fringing field approximation expression is given by [93][94]

$$w_{\text{eff}} = w \left(1 + 0.65 \frac{(1 - \frac{y(L)g}{w})g}{w}\right)$$  \hspace{1cm} (3.11)
3.4 Generalized closed-form model

Adding this fringing field effect (3.11) into equation (3.8), it now becomes

\[
V = \sqrt{\frac{12EIy(L)}{\varepsilon_0 w_{\text{eff}} \int_{L-aL}^L x^2(3L - x) \, dx}}
\]

(3.12)

An analysis of the effect of the fringing field on pull-in position is shown in Figure 3.10. The \(w/g\) ratio variation from 16 to 2 causes 5% variation in pull-in position. It is shown that if the beam is very narrow or the gap is very large (small \(w/g\) ratio), the normalized pull-in position is increased up to 0.475 or higher.

3.4 Generalized closed-form model

As described in the previous section, equation (3.10) is a good approximation to detect the pull-in position \(y_{\text{pull}}\) of the cantilever beam with a partial electrode. Using the value for \(y_{\text{pull}}\) obtained from equation (3.10) in equation (3.12), the pull-in voltage \(V_{\text{pull}}\) can be found. Solving equation (3.12) numerically is not a big challenge in Matlab or any other programming language (C, Fortran, etc.). However, a closed-form expression is desirable to reduce calculation time, simplify the calculation and give a quick analytical insight into the influence of design parameters on the cantilever pull-in.

With the FEM extracted shape function, it is difficult to find an explicit solution for the integral term in equation (3.12). On the other hand, this integral can be accurately approximated by using a Simpson’s 3/8 rule, which is given by [102]

\[
\int_a^b f(x) \, dx = \frac{b - a}{8} \left[ f(a) + 3f \left( \frac{2a + b}{3} \right) + 3f \left( \frac{a + 2b}{3} \right) + f(b) \right]
\]

(3.13)

The error of this approximation depends on the function \(f(x)\) and the length of
interval \([a, b]\). Accordingly, higher error is expected as the electrode length increases.

Applying the Simpson’s 3/8 rule, equation (3.13), in equation (3.12) as the interval \(b - a\) equals to electrode length \(l_e\), or \(a = L - \alpha L\) and \(b = L\). In addition with \(y_{PF}\) obtained from equation (3.10), a closed-form expression for the pull-in voltage of a cantilever beam with partially distributed electrostatic force can be found as:

\[
V_{PI} = \sqrt{\frac{12EIy_{PF}}{\varepsilon_\omega w_{eff} \alpha L}} \left[ f(L) + 3f \left( \frac{3L - 2\alpha L}{3} \right) + 3f \left( \frac{3L - 2\alpha L}{3} \right) + f(L - \alpha L) \right] \\
(3.14)
\]

where

\[
y_{PF} = g(0.3333 + 0.2125\alpha + 0.0623\alpha^2 - 0.3315\alpha^3 + 0.1672\alpha^4)
\]

\[
w_{eff} = w \left( 1 + 0.65 \frac{(1 - y(L)/g)g}{w} \right)
\]

\[
f(x) = \frac{x^2(3L - x)}{(g - y(x))^2}
\]

\[
y(x) = y_{PF} \left( 2.56 - \frac{16.127}{4(x/L + 0.00185)^2 + 6.2786} \right)
\]

The proposed closed-form model (3.14) of pull-in voltage is validated and discussed further in the next section.

### 3.5 Model validation and discussions

Figure 3.11 shows a comparison of the new closed-form model developed in this work with FEM simulation using CoventorWare and the model presented in [94] for various lengths of electrode. The comparison shows that the proposed model
3.5 Model validation and discussions

![Comparison of pull-in voltage](image)

\[ E=57\text{GPa}, \ v=0.33 \]
\[ L=200\mu\text{m}, \ w=50\mu\text{m} \]
\[ t=3\mu\text{m}, \ g=2\mu\text{m} \]

Figure 3.11: Pull-in voltage comparison for difference electrode length.

agrees very well with the data obtained by FEM simulation. Both models converge when the electrode length becomes shorter. However, at the values of \( \alpha \) from 0.6 to 1, the model presented in [94] deviates from the FEM simulation. The pull-in voltage gradually reduces as the electrode length increases from 0.6 to 1, while it is predicted to increase slightly in the model in [94]. A similar effect was observed in the previous section of this work and was attributed to linear effective stiffness analysis. Hence, the calculation of effective stiffness based on uniform load which was employed in [94] is believed to be the cause of this deviation.

The error found from when comparing this model to FEM simulation results is due to the fringing 3D effect and the error of shape function as shown in Figure 3.3a. However, the trend of pull-in voltage versus \( \alpha \) is predicted very well with developed model in (3.14).

The influence of the beam length on the pull-in voltage at several partial loads is shown in Figure 3.12. It is noted that the pull-in voltage does not increase
3.5 Model validation and discussions

Figure 3.12: Pull-in voltage at different beam length for various $\alpha$ using equation (3.14).

significantly as the partial electrode size is reduced from full length ($\alpha=1$) to half length ($\alpha=0.5$). Figure 3.11 in accordance with Figure 3.12 could provide a useful insight for the initial design phase of MEMS device. For electrode lengths from 0.5 to 1 of the beam length, the pull-in voltage does not change significantly. The pull-in voltage increases significantly for $\alpha < 0.5$.

Three cases of cantilever beams with wide beam and full electrode are compared with the results obtained from the new model are shown in Figure 3.13. An excellent agreement is found with the previous models [93][94][95]. The values determined by the new model developed in this work are within 1.7% deviation from the FEM results published in [93].

Table 3.1 shows several case studies for cantilever beams. A comparison of the results from the new model and the results published in [93][94][95] for Table 3.1 are shown in Figure 3.14. Cases 4 and 5 are not available for Ref. [93] and [95] as
3.5 Model validation and discussions

Figure 3.13: Comparison of pull-in voltage calculated using various models. A wide width beam full length electrode with common parameters: $E = 169$ GPa, $w = 50\mu m$, $t = 3\mu m$, $g = 1\mu m$. Case 1 ($L=100\mu m$, $\nu=0.06$). Case 2 ($L=100\mu m$, $\nu=0.32$). Case 3 ($L=150\mu m$, $\nu=0.06$).

Table 3.1: Five cases for comparison in Figure 3.14 of cantilever beams (wide and narrow beam, partial electrode), with common parameters $E = 77$ GPa, $\nu=0.33$

<table>
<thead>
<tr>
<th>Case</th>
<th>$L(\mu m)$</th>
<th>$w(\mu m)$</th>
<th>$t(\mu m)$</th>
<th>$g(\mu m)$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>300</td>
<td>50</td>
<td>1</td>
<td>2.5</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
<td>50</td>
<td>1</td>
<td>2.5</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>0.5</td>
<td>1</td>
<td>2.5</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
<td>50</td>
<td>1</td>
<td>2.5</td>
<td>0.5</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>50</td>
<td>1</td>
<td>2.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

these models are only for a full length electrode configuration.

Excellent agreement between the new closed-form model and the CoventorWare data is found in case 1. In case 2, the new model predicts similar results with models in [93][95]. Case 3 is an extreme case of fringing fields (narrow beam), the
3.5 Model validation and discussions

Figure 3.14: Pull-in voltage comparison of 5 cases in Table 3.1.

<table>
<thead>
<tr>
<th>Case</th>
<th>CoSolve [94]</th>
<th>Ref. [94]</th>
<th>Ref. [95]</th>
<th>Ref. [93]</th>
<th>This work</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.25</td>
<td>2.33</td>
<td>2.27</td>
<td>2.27</td>
<td>2.25</td>
</tr>
<tr>
<td>2</td>
<td>0.75</td>
<td>0.84</td>
<td>0.8188</td>
<td>0.8180</td>
<td>0.8109</td>
</tr>
<tr>
<td>3</td>
<td>1.20</td>
<td>1.33</td>
<td>1.21</td>
<td>1.23</td>
<td>1.28</td>
</tr>
<tr>
<td>4</td>
<td>2.30</td>
<td>2.07</td>
<td></td>
<td></td>
<td>2.37</td>
</tr>
<tr>
<td>5</td>
<td>4.00</td>
<td>3.83</td>
<td></td>
<td></td>
<td>3.97</td>
</tr>
</tbody>
</table>

error is 6.6% compared with FEM simulation. The model in [95], which employs an empirical formula for the capacitance between VLSI on-chip interconnects, achieves the highest accuracy for this narrow beam case. For cases 4 and 5, partial electrodes, the new model determined values are within 3% and 0.75% respectively of the FEM simulation results. From Table 3.1, it is evident that the new closed-form model results are in good agreement with the FEM results for various partial electrodes and on both wide and narrow beams.

Figure 3.15 shows the comparison of this work with the models in [94][95] for different gap to beam thickness ratios. There is negligible difference between the pull-in voltages determined by this work and the model presented in [95]. As the ratio increases, the results predicted by model [94] begin to deviate from the other two models.

The shape function \( y(x) \) used in (3.14), which was extracted from FEM simula-
3.5 Model validation and discussions

Figure 3.15: Pull-in voltage comparison for different ratio of gap space to the beam thickness \((g/t)\).

The beam shape function is closer to Cosine function (3.9b) as \(\alpha\) closes to 0, as seen in Figure 3.3. Therefore in the case that \(\alpha\) is small, the Cosine function (3.9b) can be used for \(y(x)\) in (3.14) for a higher accuracy result.

Furthermore, the method to derive (3.14) can be employed for any other cases that have different beam shapes under electrostatic actuation. If the beam shape function is known, a normalized pull-in position \(y_{PI}/g\) similar to (3.10) can be found. Then, equation (3.14) is changed accordingly for the closed-form solution.

For narrow beams with extreme fringing field effects, higher errors relative to FEM results are found in this work compared to the model in [95]. This is believed to be due to the employment of an approximation of the effects of fringing fields in the system as a strip line conductor with zero thickness over infinite ground plane [93]. A higher approximation of fringing field effects, i.e. the empirical formula used
3.6 Summary

in [95], might be included in the future work to improve the accuracy.

3.6 Summary

This chapter presents a new approach for the pull-in analysis of micro cantilever beams subjected to partial electrostatic actuation. A novel generalized closed-form models for the pull-in instability position and pull-in voltage of a cantilever beam with partial overlap of the beam and fixed actuation electrode is developed. The pull-in position is numerically calculated and interpolated to obtain an empirical formula for normalized pull-in position as a function of partial electrode length. The closed-form pull-in voltage expression is derived by employing Simpson’s 3/8 rule for an implicit integral. The model accounts for non-uniform distributed electrostatic force, partial electrode configuration and fringing field effects. The model is solely an analytical expression with no compensation factors required as have been used in previous works.

It is found that the pull-in stability position is independent with the beam structure, but varies with the electrode length. At full electrode length, the nominal pull-in position is around 0.44 while it reduced to 0.33 at short electrode positioned near the tip end of the beam. The analysis shows that with the variation of the pull-in voltage and pull-in position are very small as the electrode length varies from half to full length of the beam.

The developed model exhibits high accuracy for the pull-in voltage calculation in wide and narrow beams with partial electrode overlap. This is the first research work focused on analysing both pull-in position and pull-in voltage on cantilever with partial electrode. The accuracy of the model is validated through comparison with Finite Element Method (FEM) simulation results and correlation to within 6.6% error is obtained. The model is also validated against other existing empirical and analytical models and is demonstrated to be of equal accuracy with a more generalized model framework.
Chapter 4

Design to avoid pull-in instability of MEMS ohmic switches

There are no facts, only interpretations
Friedrich Nietzsche, German philosopher, (1844-1900)

Pull-in Instability is one of the key issues affecting the lifetime of the ohmic switch. This chapter presents a thorough analysis of the pull-in instability phenomenon and develops guidelines for the practical design of MEMS ohmic switches without pull-in stability issues. The proposed closed-form analytical model is then validated by FEM simulation results with high accuracy. The proposed design approach is intended to reduce the contact velocity; and subsequently to improve the contact reliability factor.

4.1 Introduction

In general electrostatically actuated MEMS structures, especially sensors, the pull-in instability phenomenon restricts the stability of travel range of the movable structures. For MEMS ohmic switches in particular, the pull-in instability phenomenon leads to a very high contact velocity that creates high impact forces between the top
and the bottom metal contacts. Pitting, hardening and some other types of contact
degradation occur faster at higher impact force and this can significantly affect the
contact resistance and lead to contact failure (sudden increase in the resistance).
This metal contact interaction mechanism is not well understood and is currently
under investigation by many industrial and academic research groups [79][80].

The contact reliability depends strongly on the contact metals used and can be
improved by several methods. Designing a MEMS ohmic switch without pull-in
instability during actuation is one possible option to reduce the switching contact
velocity and thus reduce the high impact forces that are detrimental to the device
reliability. The main objective of this chapter is to provide a thorough analysis and
derive guidelines for the design of MEMS ohmic switches without pull-in stability
issues.

To extend the stable range of travel so as to achieve full range (without pull-in
instability), several methods have been demonstrated in the MEMS field:

- Geometry leverage: Leveraged bending effect can be used to achieve full gap
  travel [87][103]. The method requires longer cantilever beam or should be paid
  by increasing actuation voltage.
- Addition of a constant capacitor: An additional series capacitor is employed
to provide stabilizing negative feedback thus extending the effective electrical
  gap of the MEMS device [104].
- Switched-capacitor: The amount of charge on an electrostatic actuator is con­
trolled via a number of switched-capacitors for increasing the stable range of
  motion [105].
- Current driving: The charge level on the actuator is controlled by means of
current driving to achieve the required amount of charge to fix the position of
  the movable plate [106].
- Tailoring the actuation waveform: The actuation is not a single step function
  but comprises two pulses. First a short pulse provides the necessary amount
  of energy to actuate the structure to the contact with near-zero velocity. The
second pulse is then applied upon contact to keep the switch from opening.

All five methods have been proved to mitigate the pull-in instability issues to a certain extent. Only the first method relates to mechanical design, the other remaining methods relate to electrical parameters and additional circuits are generally needed. The first four methods are used for applications where the stable range is an important factor such as sensors.

For MEAIS switches, the stable range is not important but minimizing the contact velocity is critical for reliable operation. Therefore the first and last methods (leverage and controlling actuation waveform) are more applicable to MEMS switch applications. Tailoring the actuation waveform to reduce the impact force is a control process not considered at the device design stage and will be considered in Chapter 6 of this dissertation.

In fact, if the tip-to-contact gap (or tip-gap) is within the stability region, the pull-in instability is prevented. This type of design was found on several fabrications, however, the analytical method for switch design was not disclosed. There are two methods that can be used to design a MEMS switch without the pull-in instability:

- **Leveraged bending method** [87][103] is based on applying electrostatic force to only a portion of the cantilever beam close to the beam’s support. The rest of the beam is used as a "lever". The tip of the beam can move the entire gap distance, even though the actuated portion only moves less than a half of the gap.

  - **Advantage**: High initial tip to gap distance, hence high isolation for RF signal
  - **Disadvantage**: High actuation voltage

- **Reduce tip-to-contact gap** (tip-gap) by adding a dimple on the tip or heightening the pad level (see Figure 2.1 Chapter 2). The tip-gap is designed
4.2 Design analysis of electrostatic MEMS cantilever switch without pull-in instability

to be smaller or equal to the displacement for the pull-in instability, therefore the switch closes before the pull-in instability appears. Unlike, the leveraged bending design the electrode can be moved to the free end of the beam thus reducing the actuation voltage.

- **Advantage**: Low voltage actuation
- **Disadvantage**: Low initial tip gap might reduce the isolation factor

It is seen that each method has its merits and limitations. The combination of both methods in one design could yield a combination of the merits of both while overcoming the inherent limitations.

In this Chapter, a design process to avoid the pull-in instability in cantilever MEMS switches is considered. The optimized length of the leveraged bending element and the tip-gap will be evaluated systematically for the first time.

The remainder of the chapter is organized as follows: An analysis and development of design without pull-in instability is described in Section 4.2. FEM validations on the developed analytical are presented Section 4.3.

4.2 Design analysis of electrostatic MEMS cantilever switch without pull-in instability

Figure 4.1 shows a cross-sectional view of a cantilever based MEMS ohmic switch. The structure is similar to the structures shown in Figure 3.1 and 3.2 in Chapter 3 with an addition of the $L_2$ portion, dimple and contact pad. When the actuation voltage is applied to the electrode, electrostatic fields at the gap between the beam and the electrode generate an electrostatic force along the beam length, $l_c$. This causes a vertical beam displacement as shown in Figure 4.2 where the dimple is neglected.

As the mass of the beam is considered zero, and there is no electrostatic force applied on the $L_2$ beam section, the BC segment (shown in Figure 4.2) is only
4.2 Design analysis of electrostatic MEMS cantilever switch without pull-in instability

Figure 4.1: Cross-sectional view of a typical cantilever beam switch design.

Figure 4.2: Superposition analysis of cantilever beam deflection under partial electrostatic force, with dimple neglected.

acting like a lever. Therefore the analysis of the cantilever structure in Figure 3.2 can be used for the cantilever beam switch shown in Figure 4.2 for the AB segment, where A is anchor point and B is the edge point of overlapped area with the drive
4.2 Design analysis of electrostatic MEMS cantilever switch without pull-in instability

electrode. Equation (3.14) can be used to calculate the pull-in voltage whereas the pull-in position is discussed below.

As discussed in the Chapter 3, the shape function of the beam portion $L_1$, can be approximated by equation (3.9c). It can be rewritten for this case as

$$y(x) = \left(2.56 - \frac{16.127}{4(x/L + 0.00185)^2 + 6.2786}\right)y_B$$  \hspace{1cm} (4.1)

where $y_B$ is the deflection at point B.

Integrating equation (4.1), the angle $\varphi$ at point B can be found as $\varphi = y'(B)$. The value of $\varphi$ is simplified, with error less than 0.01%, as:

$$\varphi = \frac{1.22}{L_1}y_B$$  \hspace{1cm} (4.2)

In MEMS structures, the deflection $y_B$ is usually very small (less than 1$\mu$m) compared with the beam length (tens to hundreds of $\mu$m), hence $\tan(\varphi) \sim \varphi$. The deflection of BC segment is approximated as

$$h_{BC} = \tan(\varphi)L_2 = 1.22\frac{L_2}{L_1}y_B$$  \hspace{1cm} (4.3)

The total deflection at the tip of the beam (point C) is

$$y_C = y_B + h_{BC} = y_B\left(1 + 1.22\frac{L_2}{L_1}\right)$$  \hspace{1cm} (4.4)

The analysis of the pull-in instability position at the tip of the beam in Chapter 3 can be used here to calculate the instability position at point B of the beam in Figure 4.2. Neglecting the BC segment, from equation (3.10) in Chapter 3 the pull-in instability position of point B can be found as

$$y_{B, PI} = g(0.3333 + 0.2125\alpha_1 + 0.0623\alpha_1^2 - 0.3315\alpha_1^3 + 0.1672\alpha_1^4)$$  \hspace{1cm} (4.5)
4.2 Design analysis of electrostatic MEMS cantilever switch without pull-in instability

with $\alpha_1 = L_c/L_1$

As established in Chapter 3, the normalized instability position at point B varies from 0.33 to 0.44 depending on the value of $\alpha_1$. Substituting (4.5) into (4.4), the total deflection at the tip of the beam, where the pull-in instability occurs, is

$$y_{C.PI} = g(0.3333 + 0.2125\alpha_1 + 0.0623\alpha_1^2 - 0.3315\alpha_1^3 + 0.1672\alpha_1^4)(1 + 1.22\frac{L_2}{L_1})$$ (4.6)

If the pull-in tip deflection predicted by (4.6) is less than the tip-gap distance, the pull-in instability appears before the contact dimple touches the contact pad. To avoid the pull-in instability the tip deflection at pull-in instability should be at least equal or larger than the total gap, or $y_{C.PI} \geq g$. The following condition should be satisfied to prevent the pull-in instability issue for Figure 4.2.

$$(0.3333 + 0.2125\alpha_1 + 0.0623\alpha_1^2 - 0.3315\alpha_1^3 + 0.1672\alpha_1^4)(1 + 1.22\frac{L_2}{L_1}) \geq 1$$ (4.7)

In practice, the dimple is usually added to achieve contact without the pull-in instability with a shorter beam length as shown in Figure 4.1. Another dimple design purpose is to add another material other than the beam material to improve certain design objectives. For example, a gold beam has good resistivity but very low hardness and melting point which reduce the reliability of the contact, Table 2.6. A harder material like Ruthenium can be used as a dimple to improve the contact reliability under impact force. For that reason, the tip-gap, $t_g$, is reduced as $t_g = g - h_{dimple}$. Then condition (4.7) should also be changed according for practical dimple added design as

$$t_g = g - h_{dimple} \leq (0.3333 + 0.2125\alpha_1 + 0.0623\alpha_1^2 - 0.3315\alpha_1^3 + 0.1672\alpha_1^4)(1 + 1.22\frac{L_2}{L_1})g$$ (4.8)
Based on the analysis shown in Figure 3.11, the value of \( \alpha_1 \) must vary within 0.5 to 1 to achieve a low actuation voltage and high deflection at pull-in. Therefore, with the goal of reducing actuation voltage, \( \alpha_1 \) is designed within this range. Then, the normalized pull-in position does not change significantly (around 0.44), hence the condition from (4.8) can be simplified as

\[
\tau_g = g - h_{\text{dimple}} \leq 0.44g(1 + 1.22\frac{L_2}{L_1})
\] (4.9)

Equation (4.9) shows the relationship between the tip gap and the dimple height versus the ratio of \( L_2/L_1 \) for a no pull-in design. It is shown that if the ratio \( L_2/L_1 \geq 1.05 \), the dimple is not necessary to be added to achieve no pull-in behaviour for full range tip gap. For higher accuracy requirement and for the cases that \( \alpha_1 < 0.5 \), equation (4.8) should be used for a better estimation.

These above analysis can be used as design guidelines for cantilever MEMS switches operating without the pull-in instability. FEM simulations are used in the following section to validate the developed analytical method.

4.3 FEM comparison

Figure 4.3 shows a comparison between CoventorWare FEM simulations and analytical calculations for the structure described in the figure caption. Here, \( L_1 \) is fixed at 100\( \mu \)m, \( \alpha_1 = 1 \). The length of the "lever" \( L_2 \) is varied from 0 to 180\( \mu \)m. It is shown that the FEM simulations are in good agreement with the calculations in both pull-in voltage and tip deflection at pull-in. The imperfect FEM evaluation trend is due to the resolution of the voltage step in the pull-in detection on CoSolve FEA. The normalized pull-in position increases linearly from 0.44 to 1 as \( L_2 \) increases from 0 to 100\( \mu \)m. As \( L_2 \) goes beyond 100\( \mu \)m (or \( L_2/L_1 > 1 \)), the tip touches to the contact pad before the pull-in instability occurs. Figure 4.3 also shows that as long as the tip deflection at pull-in is within the gap range, the length of \( L_2 \) has no effect on the pull-in voltage.
Figure 4.3: Coventor FEM simulation (markers) and calculation (lines) of pull-in voltage (right axis) and travel distance (left axis) versus length of "lever" $L_2$ (for $L_1=100\mu m$, $\varphi_1=1$, $E=77\text{GPa}$, $t=1\mu m$, $g=2.5\mu m$, $w=50\mu m$, $h_{\text{dimple}}=0\mu m$).

The pull-in voltage is slightly reduced as $L_2/L_1 > 1$ because the switch is closed before instability. The longer the length of $L_2$, the shorter the deflection of point B as seen in Figure 4.3. Applying the known point B deflection as $y_{\text{pull}}$ into equation (3.14) in Chapter 3, the pull-in voltage can be predicted. The trend agrees well with the FEM validation as shown in Figure 4.3.

Figure 4.3 shows an additional FEM simulation for the case of $\varphi_1=0.5$. It is seen that the tip deflection at pull-in does not change significantly as $\varphi_1$ is reduced from 1 to 0.5. This result has been previously predicted by observing Figure 3.7 in Chapter 3.

To avoid the pull-in instability with a short length of $L_2$, the dimple is added with the height ($h_{\text{dimple}}$) to satisfy the condition: tip deflection at pull-in + dimple height $\geq$ total gap. For example, for the structure shown in Figure 4.4 with $L_2=20\mu m$, the dimple height should be designed at the shortest of $h_{\text{dimple}} = \ldots$
4.3 FEM comparison

Figure 4.4: Coventor FEM simulation (markers) and calculation (line) of pull-in travel distance versus length of "lever" $L_2$ for two cases of $\varphi_1=1$ and $\varphi_1=0.5$. (for fixed $L_1=100\mu$m, $E=77$ GPa, $t=1\mu$m, $g=2.5\mu$m, $w=50\mu$m, $h_{\text{dimple}}=0\mu$m). It shows the minimum $h_{\text{dimple}}$ should be designed to avoid pull-in instability at $L_2=20\mu$m to avoid pull-in instability issue.

Figure 4.5 shows the trade-off between pull-in voltage and travel distance for various ratios of $L_1/L$ with the total length ($L$) fixed, $\varphi_1=1$. For larger $L_1/L$, the pull-in voltage is reduced, but the tip deflection at pull-in is increased. The prediction of equations (3.14) and (4.9) are validated by the FEM results. It is seen that the optimal point would be at $L_1/L=0.5$. The full gap travel is achieved while minimizing the actuation voltage. The actuation voltage can be reduced by increasing $L_1$, however, a dimple should be added to avoid the pull-in instability. Figure 4.5 shows a clear guideline for engineers on designing MEMS switches (or any other general MEMS structures) operating without pull-in instability. The design method helps to reduce the contact velocity by removing the pull-in instability, therefore it is believed to improve the contact reliability of MEMS switches.
4.4 Summary

In this chapter, an analytical approach to design of a MEMS cantilever switch without the pull-in instability is presented. Following this guideline, a switch can be designed such that it will close contact before the instability occurs. This in turn will translate into a reduced contact speed and force. The results based on the proposed analytical method were validated and found to agree well with FEM simulation results. The proposed guidelines can be instrumental in general MEMS devices design where the stable range of travel is an important factor. The analytical closed-form model for leveraged bending method (4.9) assists the design of switches with a full travel range without suffering instability. It is shown that if the electrode is positioned near the anchor with the length equals to half of beam length, the cantilever structure can travel the full gap without instability effect. However, the full range travel is paid off by the pull-in voltage. The optimal design can be found...
by plotting a combined graph which shows pull-in voltage and tip deflection at pull-in which a relatively fast and straightforward procedure form a MEMS designer’s prospective.

Furthermore, the developed analytical method can also be applied for other electrostatic MEMS designs, such as sensors, where the travel distance is an important parameter.
Chapter 5

Dynamic model for contact mechanics and model-based analysis of switch degradation effects

Science, in the very act of solving problems, creates more of them.

Abraham Flexner, American educator (1866-1959)

This chapter presents a novel nonlinear dynamic model of an ohmic MEMS switch to investigate the switch deformation, bouncing behaviour and contact mechanics. The model is based on dynamic Euler-Bernoulli beam theory and take into account the effect of squeeze-film damping and contact mechanics interaction. A low-complexity formulation based on Finite Difference Method (FDM) is employed to solve the model equations in the time domain.

A multi-asperity contact model which accounts for fully elastic, rough surfaces with adhesion is developed. The dynamic model is integrated into the full 2-D switch model as a boundary condition for contact interaction. The contact resistance is modeled through the dynamic interference of the beam into the contact.
The proposed methodology is validated using fabricated single-pole single-throw (SPST) ruthenium contact radio frequency (RF) MEMS switch. This contribution is intended to provide an accurate yet reasonably uncomplicated analytical approach for analysis of MEMS switch contact behaviour and switch degradation effects.

5.1 Introduction

When a sufficiently large actuation voltage is applied, the resultant electrostatic force pulls the switch armature to the closed position on the contact pad. With a simple step actuation voltage, the electrostatic force increases continuously as the contacts approach each other and the beam approaches the contact at a considerable speed, e.g. 0.38m/s for switch presented in [110]. Especially, for the switch in which the pull-in instability is present, this effect results in higher impact forces. Under the high impact between the tip and the contact pad, the switch rebounds several times before making permanent contact. It has been experimentally observed in many works that ohmic switches bounce several times before making permanent contact [111][112][113][114].

The bouncing behaviour increases the effective closing time of the switch. The high contact force is believed to be the main contributor to mechanical failure of the switch. This instantaneous impact induces local hardening or pitting of contact materials [115]. Accumulation of these effects results in two major failure modes: stiction, which causes fail-to-open malfunction and resistance degradation, which increase the insertion loss to the RF signal. These dynamic behaviour are significant factors affecting the life-time of the switch. Several experimental results report that adhesion, welding, melting and contact resistance have been found which may lead to the switch failure [81][115][116][117]. Contact resistance tests were done at Northeastern University [81] on gold-on-gold contact switches which showed that, in the early life, contact resistance reduced. However, resistance increased progressively after $10^6$ cycles. Switches have eventually failed in by either high resistance or adhesion. A shorter lifetime was observed on hot switching devices.
5.1 Introduction

The failure mechanism which relates to contact bounce and the impact force of the contact during operation is not fully understood as the underlying physics is extremely complex. Reducing the bouncing effect is crucial not only for shortening the switching time but also to improve reliability and performance. In order to achieve these goals a deeper understanding of the physics of micro level contact interaction is required. Therefore, it is fundamentally important to thoroughly analyse this dynamic characteristic of the vibro-impact phenomenon. A development of a full dynamic model accounting for contact mechanics is necessary to investigate this behaviour.

A dynamic contact model is required in the system model to interact with the beam during impact. It would allow to accurately capture the bouncing behaviour of the switch. It would also be useful in understanding failure mechanisms and assist choosing most efficient approaches for improving the reliability of the device. A lot of research has been undertaken on this subject. The first analysis of elastic contact of two bodies without adhesion was well developed by Hertz in 1882 [118]. The contact area and deformation models developed by Hertz have been widely adopted. The JKR and DMT models are two successful extensions of the Hertz model to include contact adhesion. The Johnson, Kendall, and Roberts (JKR) model [119] assumes that adhesion only exists inside the contact area, therefore both compressive and tensile stresses can only exist in the contact area. Another model, by Derjaguin, Muller, and Toporov (DMT) [120], adds attractive interactions outside the area of contact but do not deform the profile. The contact profiles are assumed to be identical as in Hertzian contact.

A simple analytical model of elasto-plastic and fully plastic deformation with adhesion was developed by Majumder et al. [81]. The model was developed based on the assumption that only elastic deformation may occur on both loading and unloading. Kogut et al. [121] developed a finite element model (FEM) to study elastic-plastic contact between sphere and a flat surface with adhesion. A simple curve-fit equation of force vs. displacement and contact area was developed. The details of unloading for the residual radius of curvature and residual interference after the sphere is completely unloaded without adhesion were described in [122].
The work by Kadin et al. [123] found that plastic deformation does not only occur for the first cycle of unloading but also the subsequent unloadings.

A Molecular Dynamics (MD) simulation method was introduced in 2006 by Song and Srolovitz from Princeton University [124]. This method describes the interaction of individual atoms in nano-scale structures. It is found that the magnitude of plasticity is positively proportional with adhesion energy during unloading. The movement of atoms occurs mostly from the side with the bump to the flat substrate, whereas a very small number of atoms transfer from the flat side to the bump.

Some of the methods mentioned above accurately capture contact behaviour. However they only consider static contact interaction (stress-strain curve). For the dynamic interaction under the switching conditions of MEMS switches, more attention should be paid to the integration of many other factors such as: squeezed-film damping, electrostatic force, beam structure to achieve a full dynamic MEMS model.

In MEMS switches, in order to reduce the actuation voltage, the distance between beam structure and substrate is minimized, whereas the area of the electrode is maximized in MEMS switches. Under such conditions, the so-called squeezed-film damping is a pronounced effect. Under actuation, the beam creates a movement of the fluid (air) underneath and the movement is resisted by the viscosity of the fluid. The pressure is distributed underneath the beam and may act as a spring and/or damping forces. Recent studies show that the damping force dominates the spring forces at low mechanical frequency, while the spring force dominates over the damping force at high mechanical frequency of the beam [125].

Accurate dynamic modeling of electrostatic MEMS switches structures is very challenging due to the mechanical-electrical coupling effect and the nonlinearity of the structure, electrostatic force and air damping effects. A number of models have been developed to investigate the dynamic operation of the switch and contact behaviour. Some review papers [126][127] present an overview of the existing models for the electrostatically actuated MEMS. These review papers summarise the fundamental knowledge on the nonlinear behaviour of the switch, including beam theory,
damping effect, contact interaction, etc. In more detail, McCarthy et al. [88] presented a dynamic model which is a one-dimensional finite difference dynamic model based on Euler-Bernoulli beam theory, squeeze-film damping and a linear contact spring to simulate the dynamics of an ohmic contact RF MEMS switch both before and after contact. The work included the bouncing features of the switch but the contact mechanics behaviour such as (deformation, adhesion) were not taken into account.

The dynamics and squeeze-film damping was calculated based on a lumped model of a capacitive RF MEMS switch in [128]. The dynamic behaviour of a MEMS switch using finite element method with electrostatic actuation was studied in [113]. The squeezed-film damping and contact interaction were not taken into account in those studies.

A number of works by Decuzzi et al. [129][130] reported a model based on Euler-Bernoulli theory and theoretical bouncing dynamics of a resistive switch. A van der Waals term and linear spring were employed to represent the interaction forces (attractive and repulsive) between contacts. A dynamic fourth-order beam deflection equation model with the effect of the dynamics of the switch on its opening time was presented in [131]. That model included the adhesion force due to van der Waals type forces and metal-to-metal bonds.

With its multi-physics capability, the Finite Element Method (FEM) is one of the most comprehensive tools for MEMS switch modeling and analysis. In recent years, a lot of research work employing FEM to develop 3-D models for MEMS switches has been reported. A 3-D model of a micro-switch including structure and electrostatic actuation but without mechanical contact and squeezed-film damping was developed in [90]. The model does not include contact as the focus of the work was to remove contact bouncing. Guo et al. [66] implemented a purely elastic contact behaviour with the JKR adhesion model integrated with Euler-Bernoulli beam theory using a finite element method in ANSYS. Lishchynska et al. [132] employed linear and piece-wise linear system identification techniques to interpret data extracted from a 3-D ANSYS model of the switch. Air damping and contact mechanics were not considered by the authors.
A brief literature review shows that the beam structure and electrostatic actuation can be accurately modeled by FD or FE methods, especially in commercialized software like ANSYS. However, methods for squeezed-film damping and contact interaction modeling are not readily available. It is apparent that the majority of dynamic microswitch models proposed to date do not incorporate the inherent multi-asperity rough surfaces into the overall dynamic MEMS switch model. Presented work intends to address this gap.

In this chapter, the rough surface with multi-asperity height following a Gaussian distribution is employed into a nonlinear dynamic model of an electrostatic ohmic-contact switch. The model comprises dynamic Euler-Bernoulli beam theory, electrostatic force, squeeze-film damping effect, and nonlinear multi-asperity contact mechanics. The model also takes into consideration the real geometry of the switch. A time-transient finite difference analysis (FDA) [88] is used to model the dynamic behaviour of the electrostatically actuated MEMS switch.

There are a number of factors that affect the lifetime of the metal to metal contact switch such as: contact degradation, stiction, wear, plastic deformation and fatigue of the cantilever beams [133][134][72][135][136]. A number of studies have been reported on the lifetime evaluation of ohmic switches [72][135][82]. The devices were characterized in terms of current handling, contact resistance and actuation voltages during testing. A notable decrease of the actuation voltage of switches held for long periods in the actuated condition was observed [72][82]. This effect could be due to spring constant reduction, plastic deformation or a combination of these processes. An analytical method to identify degradation mechanisms in capacitive switches is reported in [137]. Little information in the literature deals with the evaluation of this effect for an ohmic switch. An analysis based on a lumped model which does not take account of the real device shape might yield misleading conclusions, while the evaluation based on a 3-D model is very time consuming.

A 2-D FDM model developed in this work is used to evaluate actuation voltage drift under stress, with time. The contributions of possible mechanisms of mechanical degradation of the switch to the measured change in performance are determined. The correlation found in simulation is used to create a model relating actuation stress
5.2 Model description

Figure 5.1: Cross-sectional view of a typical cantilever beam switch design.

to eventual failure. Follow that, the model to predict the failure point of a device based on the correlation trend found is presented. The evaluation of the maximum peak stress permissible to meet the lifetime criterion is demonstrated.

It is foreseen that the proposed approach will be instrumental in providing a better insight into the reliability of MEMS switches and will, ultimately, provide a basis for developing and implementing control strategies to maximize their lifetime.

The remainder of the chapter is organized as follows: Section 5.2 presents the development of the dynamic model, Section 5.3 describes the MEMS ohmic contact switch used to validate the model. Simulation and experiment results comparisons are showed in Section 5.4.

5.2 Model description

Figure 5.1 shows the cross-sectional view of a typical ohmic contact-type RF MEMS micro-switch. The development of the Finite Difference Model (FDM) can be divided into four components. The main part is the beam which is modeled by dynamic Euler-Bernoulli beam theory. It is combined with the electrostatic force equation, squeezed-film damping formulation and finally the contact boundary conditions.
5.2 Model description

5.2.1 Beam deflection formulation

The modified beam is modeled using Euler-Bernoulli theory with a fixed cross-sectional area along the length of the beam (Figure 5.1). The dynamic beam equation consists of electrostatic force equation and squeeze-film damping formulation. The equation of motion is

\[-EI \frac{\partial^4 y}{\partial x^4} + f_e - P_d = m\frac{\partial^2 y}{\partial t^2}\]  

(5.1)

where \(E\) is effective Young’s modulus, \(I\) is the second moment of the cross-sectional area, \(y(x,t)\) is the downward deflection of the beam, \(f_e\) is the electrostatic force per unit length, \(P_d\) is the damping force per unit length, and \(m\) is the mass per unit length of the beam.

Due to nonlinearities of electrostatic force and the squeezed-film damping (described in the next sections), an analytical solution is impractical and a finite difference solution is sought. The one-dimensional time-transient finite difference method studied in [88] is used to solve the governing equation (5.1). The accuracy of the method is based on the chosen scale of the discrete elements and the time steps.

The standard central difference approximations for \(\frac{\partial^4 y}{\partial x^4}\) and \(\frac{\partial^2 y}{\partial t^2}\) are employed. The four spatial boundary conditions are necessary to find a solution. The clamped end of the beam has zero displacement and zero slope \((y(0) = 0, y'(0) = 0)\) while the free end has zero bending moment and zero shear force \((M(L) = 0, V(L) = 0)\). These conditions allow (5.1) to be expressed solely in terms of nodal deflections \((i = 1, 2, ..., N)\) as shown in Figure 5.2. The node 0, N+1, N+2 are image points needed for the finite difference approximations of the boundary conditions as

\[
\begin{align*}
y(0) &= 0 \Rightarrow y_1 = 0, \\
y'(0) &= 0 \Rightarrow y_0 = y_2, \\
M(L) &= 0 \Rightarrow y_{N+1} = 2y_N - y_{N-1}, \\
V(L) &= 0 \Rightarrow y_{N+2} = 4y_N - 4y_{N-1} + y_{N-2},
\end{align*}
\]  

(5.2)
5.2 Model description

5.2.2 Electrostatic forces

For the electrostatic actuation, it is assumed that the individual elements of the beam and gate form a parallel plate capacitor. The node which does not overlap with the electrode has zero electrostatic force. The attractive electrostatic force between nodes that overlap with the electrode area can be written as

\[ f_{ei} = \frac{\varepsilon_0 V^2 w l_i}{2(g - y_i^t)^2} \]  

(5.3)

where \( \varepsilon_0 \) is the permittivity of the free space, \( V \) is the electrical potential between beam and gate, \( w \) is width of the uniform beam, \( l_i \) is the length of the node \( i \). The superscript 't' indicates that the deflection of node \( i \), \( y_i \), is evaluated at time \( t \). Two nodes positioned above the electrode’s edge \((i_{\text{start}}, i_{\text{end}})\) are assumed to half electrostatic force of above value shown in (5.3). Fringing field effect is neglected.

5.2.3 Squeeze-film damping

At atmospheric conditions, when the beam is moving downward or upward, the air between the beam and the gate is squeezed out or in, respectively. This is called the squeeze-film damping effect. In general, the damping effect of a mechanical structure in a viscous fluid can be analysed using the Navier-Stokes equations. In the present
5.2 Model description

case where the gap between the beam and the substrate is small compared to the lateral dimension, the Reynolds equation can be used

$$\frac{\partial}{\partial x} \left( \rho h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \rho h^3 \frac{\partial p}{\partial z} \right) = 12\mu \frac{\partial (p h)}{\partial t}$$

(5.4)

where $x, z$ are position over the length and width of the beam, $p(x, z)$ is the gas film pressure in the gap per unit area, $\rho$ is the air density, $h = g - y$ is the beam-to-substrate distance and $\mu$ is the gas viscosity.

This equation (5.4) can be solved implicitly to determine the damping force at each node and time step. Due to the absence of $z$ dimension in (5.1), the $z$-dependence should be removed. Assuming that $\rho$ and $\mu$ are constant throughout the fluid and $h$ is independent of $z$, equation (5.4) can be rewritten as [88]

$$2wh^2 \frac{\partial h}{\partial x} \frac{\partial P}{\partial x} + \frac{2wh^3}{3} \frac{\partial^2 P}{\partial x^2} - \frac{8h^3}{w} P(x) = 12w \mu \frac{\partial h}{\partial t}$$

(5.5)

where $w$ is the beam width and $P(x)$ is the pressure acting on the beam axis ($z=0$).

The standard central difference approximation for $\partial P/\partial x, \partial^2 P/\partial x^2$ and $\partial h/\partial x$ are used in this formation. The damping pressure at time 't' is calculated based on the beam configuration at that time. Then, this value is used in (5.1) to calculate the beam displacement at the next time step 't+1'. Thus, the beam configuration at 't+1' can be used to find the damping pressure at 't+1'. The procedure iterates for all time steps of the simulation. Due to this requirement, the backward derivative approximation for $\partial h/\partial t$ should be employed in (5.5).

Boundary conditions should also be determined to solve (5.5). At the clamped end, the air cannot escape (blocked by the anchor) and the air pressure at the free end is the atmospheric pressure. Therefore, the boundary conditions are

$$\begin{align*}
\left. \frac{\partial P}{\partial x} \right|_{x=0} &= 0 \Rightarrow P_0 = P_2, \\
P(L) &= 0 \Rightarrow P_N = 0,
\end{align*}$$

(5.6)
5.2 Model description

Before the solutions provided in (5.5) can be substituted into (5.1), they must be converted to the force per unit length by

\[ P^*(x_i) = \frac{2w_i}{3} P(x) \]  

(5.7)

And at initial position and time, \( t=0 \), the damping pressure along the beam is set equal to zero.

5.2.4 Multi-asperity nonlinear contact with adhesion

In practice, both the surface of the beam and the substrate are rough surfaces [138]. The surface roughness is represented by asperities. Therefore, only a few asperities on both surfaces make contact. The contact between the cantilever and the drain tips are modelled as the interaction between a smooth rigid surface with a multi-asperity rough surface. Figure 5.3 shows a schematic representation of the contact interaction between cantilever beam with the drain. The assumption of Greenwood and Williamson [139] regarding the statistical distribution of the asperities with identical radius is used here. The whole height \( z \) probability density function which is assumed to be Gaussian is given by:

\[ \phi(z) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( \frac{-z^2}{2\sigma^2} \right) \]  

(5.8)

where \( \sigma \) is the standard deviation of asperity heights.

Figure 5.4 represents equation (5.8) showing the half bell curve of a Gaussian distribution of asperity heights. Following this distribution law, around 68.2% of all asperities are in the range from 0 to \( 1\sigma \), while only 4.2% of asperities have heights higher than \( 2\sigma \). As the asperities have different heights, the beam only interacts and creates deformation on several of the highest asperities. The number of asperities in contact with the beam tip is:

83
5.2 Model description

Figure 5.3: Multi-asperity interaction of contact rough surfaces.

\[ n = D A_n \int_{d}^{\infty} \phi(z) dz \]  \hspace{1cm} (5.9)

where \( D \) is the area density of the asperities, \( A_n \) is the nominal contact area.
5.2 Model description

The interference of asperity $i$ is defined as:

$$\delta_i = z_i - d$$  \hspace{1cm} (5.10)

where $d$ is the separation between the smooth surface and the mean of the rough surface height.

As discussed above, the contact is a very complex phenomenon, i.e. elastic, elasto-plastic deformation may be involved on each asperity. It is assumed that the beam dynamics are most strongly influenced by elastic contact interactions and for this model all asperity deformations beyond the plastic limit are taken as having an elastic energy transfer mechanism \[66\]. During loading, the kinetic energy of the beam is gradually transferred to the underneath contact, and stored in the elastic deformation of the asperities. The elastic strain in elastically deformed asperities acts like a spring and applies a pull-out force, $P_{out}$, to the beam. As a result, the beam behaviour, during unloading, is not only defined by the restoring moment, $M$, due to bending, but also influenced by the pull-out force, $P_{out}$, and adhesion force, $F_{ad}$, of asperities that are in contact. The influence of each individual asperity depends on its own contact interaction, $\delta_i$. The total pull-out force, $P_{out}$, and adhesion force, $F_{ad}$, are obtained by summing individual asperity contributions using a statistical equation model:

$$P_{out} = DA_n \int_{d}^{\infty} p(z - d)\phi(z)dz$$  \hspace{1cm} (5.11)

$$F_{ad} = DA_n \int_{d}^{\infty} f_{ad}(z - d)\phi(z)dz$$  \hspace{1cm} (5.12)

where $p(z-d)$ and $f_{ad}(z-d)$ are the pull-out force and adhesion force of an individual asperity.

When the beam tip approaches the contact, the beam is no longer cantilevered with free end, but is fixed at one end and contact-supported at the other end.
Therefore, the shear force described by (5.2) for boundary conditions is no longer zero but now becomes

\[ V(L) = -(P_{out} - F_{ad}) = -\frac{\partial M}{\partial x} = -\frac{E_I}{L} \frac{\partial^3 y}{\partial x^3} \]  

in which the derivation \( \frac{\partial^3 y}{\partial x^3} \) is solved by finite difference approximation.

### 5.3 Device under test

Figure 5.5(a) shows the top view of the SEM micrograph of a SPST ruthenium contact RF-MEMS switch used in this work. The prototype is provided by Analog Device Inc., Ireland with non-disclosed design and fabrication technologies. The small cross-section beam area near the anchor was designed to reduce the switch actuation voltage. Stops on either side of the beam with high contacts beneath were designed to protect the beam from collapsing to the gate. Figure 5.5(b) shows the side view of the switch. The switches were made of gold and the nominal design dimensions of the switches are as follows: beam to gate gap is 0.6\( \mu \)m, tip height is 0.26\( \mu \)m. The wide rectangular region, where the gate is positioned directly beneath has length \( \times \) width of 63 \( \times \) 38\( \mu \)m. Both small cross-section portions have length \( \times \) width of 14 \( \times \) 14\( \mu \)m. Ruthenium layers are designed to increase the hardness of the contacts. The contact area is around 1.2\( \mu \)m\(^2\). The structure looks like a membrane but force is uniform along the length and high rigidity over cross-section make it behave like a beam, e.g. no lateral deformation was observed, so is considered a beam. The designed thickness of the beam is 6\( \mu \)m. The fabrication process of the device has been described in detail in [109].

The simulation methodologies for electrostatically actuated cantilever structure developed in the previous section are applied to this switch. The beam is fixed at the anchor. The tip is free end of the cantilever. When the tip touches the underneath contact (drain), the beam is no longer fixed-free cantilever. It becomes a fixed-supported beam with boundary conditions as showed in (5.13). In total 22 nodes, each node is 4.1\( \mu \)m length, over the whole beam length and time step of 1ns.
are chosen in the model. Those values are selected based on a preliminary evaluation on compromising between accuracy and simulation running time.

The two stops have small effect on the beam stiffness, therefore they are neglected for simplification. As the model is 2-D, the beam is assumed to be rigid over the cross-section.

Notice that the switch under investigation in this work has a quite complex geometrical structure which is challenging to model using the proposed finite difference analysis (FDA). Since the beam consists of two different cross-sectional areas, four artificial nodes are required. Two imaginary nodes to the right represent the extension of the narrow beam, while two additional nodes to the left represent an imaginary extension of the wider beam. The description of the method can also be found in [88].
Nominal designed dimensions with gold material for the beam (Young’s modulus $E = 78$ GPa, density $\rho = 19300$ kg/m$^3$) were adopted in the simulation. On each time step, based on the previous position of the beam nodes, the parameters in (5.1) are calculated, then a new status of the beam is extracted. When the tip height between beam and drain is zero (i.e. the beam is in contact with the drain), the multi-asperity contact model which is discussed in Section 5.2.4 is used as a boundary condition of the governing equation (5.1).

5.4 Simulation and experiment results

5.4.1 Contact resistance

In the Section 5.2.4 the multi-asperities nonlinear contact with adhesion was developed. However, the real roughness of the contacts is unknown. The evaluation of the multi-asperities parameters is presented in this section. Unlike the static method as described in previous works [140][81] in which actuation voltage needed to be converted into contact force, this work presents the dynamic contact resistance model. The resistance can be extracted directly from the known deformation and asperities radius from full switch modeling.

Figure 5.6 shows the measured resistance of the switch under actuation voltage for 3 cycles: $9^{th}$, $84^{th}$ and $327^{th}$. The switch resistance includes contact, beam, pads and connection resistances. Assuming that the beam, pads and connection resistances are constant, the contact resistance is smaller than that measured and the change of switch resistance is the change of contact resistance. The higher the actuation voltage applied, the lower the resistance against the signal running through the switch. The figure shows that the pull-in voltage is reduced with cycles. However, this phenomenon does not affect the trend of resistance. The reduction of pull-in voltage will be discussed in the next section of this chapter. The difference between resistances at different cycles is observed. It is believed that this is due to plastic deformation (permanent deformation) during cycling combined with a change in overdrive force as shown in section 5.4.3. In more details, e.g. at 75V
5.4 Simulation and experiment results

Figure 5.6: Measured switch resistances under actuation voltages for 3 cycles.

actuation, the switch resistances are 1.46, 1.34 and 1.29 Ohm at cycle 9th, 84th and 327th respectively. This phenomenon is very complex but the reduction is small after each cycle. For simplicity, it is assumed that only elastic deformation occurs during contact interaction.

In literature, the two widely used elastic contact models with adhesion are the Johnson-Kendall-Roberts (JKR) [119] and Derjaguin-Muller-Toporov (DMT) [120] models. The JKR model is the most appropriate for a large bump, high adhesion energy, while the DMT model is best applied to small, more rigid and low adhesion contacts [141]. The dimensionless parameter which was defined by Tabor [142] can be used here to determine which model can be applicable for the switch shown in Figure 5.5.

\[ \mu_T = \left( \frac{Rw^2}{E^2z_0^3} \right)^{1/3} \]  

(5.14)

where \( R \) is the asperity radius, \( w \) is the adhesion energy, \( z_0 \) is atomic radius of
material used, $E^*$ is the Hertz elastic modulus defined by:

\[
\frac{1}{E^*} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \tag{5.15}
\]

where $E_1, E_2, \nu_1, \nu_2$ are Young’s moduli and Poisson’s ratios of the two contacting surfaces, respectively.

The switch was made of gold, but due to the low hardness characteristics of gold, ruthenium layers were electroplated on both contact surfaces to improve the reliability. As both contact surfaces are ruthenium, $E_1 = E_2 = E, \nu_1 = \nu_2$. The ruthenium (Ru) material has Young’s modulus $E=414\text{GPa}$, Poisson’s ratio $\nu=0.3$, surface energy $\gamma=4.2$, adhesion energy $w=2\gamma$ and atomic radius $z_0=0.27\text{nm}$. Substituting these value with the nominal asperity radius is assumed to be $R=88\text{nm}$ (Figure 5.7), $\mu_T=1.9$ is found. According to JKR-DMT transition work presented in [143] the model used in this switch is lying in the transition area between JKR and DMT models. Following the adhesion map which is shown in [144], the adhesion force is very close to JKR model. Therefore, for simplification the Ru-Ru contact can be assumed to be in JKR regime.

Figure 5.7 shows the SEM micrograph of the contact pad of switch used in this work. It is seen that the surface is rough with variety sizes and heights of asperities.

As previously stated, contact is a very complex phenomenon. It may involve elastic or elasto-plastic or even fully plastic deformations. The temperature, humidity or hot-switching conditions are also important factors to the operation of the contacts. But for simplicity, it is assumed that no plastic deformation occurs and the asperities recover to the original condition after unloading. According to the law of conservation and transformation of energy, during contact, the energy that the beam transferred to the bump on penetrating into the contact is now equal to the elastic strain energy stored in the asperities. With the elastic deformation of asperity $i$, $z_i - d$, elastic energy in each individual asperity reflects back to the beam like a spring with a force $p(z - d)$ used in (5.11) is

\[
p(z_i - d) = \frac{4}{3} E^* R^{1/2} (z_i - d)^{3/2} \tag{5.16}
\]
5.4 Simulation and experiment results

Figure 5.7: SEM micrograph of contact pad.

The adhesion force, $f_{ad}$, for one ruthenium-ruthenium contact as considered in this work can be calculated via the JKR contact adhesion model as

$$f_{ad} = 3\pi \gamma R$$  \hspace{1cm} (5.17)

Summing all the elastic forces and adhesion forces of all individual asperities, the total contact forces can be found. This value is then used in (5.13) for the boundary condition of the switch model.

For the given parameters of the switch described in the previous section, the multi-asperities contact models are assumed as: the nominal contact area, $A_n$, is 1.2\(\mu\)m\(^2\), the asperity density, $D$, is 20\(\mu\)m\(^{-2}\). Hence, there are totally 24 asperities in the rough surface contact. Following the Gaussian distribution assumption, the heights for 24 asperities can be roughly arranged as shown in Table 5.1.

The finite difference model (FDM) in section 5.2, which includes the JKR regime
Table 5.1: 24 asperities heights distribution

<table>
<thead>
<tr>
<th>Height (z)</th>
<th>0.5σ</th>
<th>1σ</th>
<th>1.5σ</th>
<th>2σ</th>
<th>2.5σ</th>
<th>3σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of asperities</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

contact described above, is written in M-files programming in Matlab, the scripts can be found in Appendix A. The penetration of the beam into the contact (or contact deformation) can be extracted by including the contact model into the whole switch model.

All asperities are assumed to have identical radius, $R$. The highest asperity is at $3\sigma$. At the initial phase of the contact, the beam touches the highest asperity first. The higher the actuation voltage applied, the higher the interference of beam into the rough surface. As the penetration increases the beam will make contact with the lower asperities. Therefore, at a certain value of contact force (or actuation voltage) only a few highest asperities will make contact with the beam.

Figure 5.8 shows the simulation of contact penetration, or displacement, under actuation voltages at 3 asperity radii, $R = 0.05$, $0.1$ and $0.2\mu$m. All other parameters are kept identical. It is observed that the penetration is high at low asperity radius. The roughly assumed asperities heights result in the trends of contact penetration not being smooth. At $R=0.05\mu$m, only the 6 highest asperities are in contact with the beam at an actuation voltage of 80V.

Figure 5.9 shows a similar simulation for three different standard deviations of the asperities heights, $\sigma=2$, $4$ and $8$nm. It shows that the penetration is smaller at lower standard deviation. The smaller difference between asperities heights due to small values of makes the transition between asperities more smooth, therefore the penetration is smoother than the cases of higher $\sigma$.

The interference of asperity $i$ can be found from the penetration of the beam into contact, $\delta$, as

$$\delta_i = z_i - (3\sigma - \delta) \quad (5.18)$$
5.4 Simulation and experiment results

Figure 5.8: Simulation of contact penetration of Ru-Ru contact under actuation voltages at 3 asperity radii.

Figure 5.9: Simulation of contact penetration of Ru-Ru contact under actuation voltages at 3 standard deviations.

If $\delta_i$ is higher than zero ($\delta_i > 0$), the asperity has a physical contact with the
beam. The contact radius of that asperity with the smooth surface (the beam) is

\[ r_i = \sqrt{\delta_i R} \]  \hspace{1cm} (5.19)

The resistance of contact asperities can be found by [81]

\[ R_i = \frac{4p l_e}{3\pi r_i^2} + \beta \frac{l_e}{r_i} \frac{\rho}{2r_i} \]  \hspace{1cm} (5.20)

where \( \rho \) is material resistivity, \( l_e \) is electron mean free path of the material, \( \beta \) is a slowly varying function of the ratio \( l_e/r_i \), with \( \beta(0)=1 \), and \( \beta(\infty)=0.694 \).

Function (5.20) accounts for the full range of ratios between contact radius and the electron mean free path of the material used.

Assuming that all contact spots are independent and conduct in parallel, the contact resistance, \( R_c \), can be found from

\[ \frac{1}{R_c} = \sum_i \frac{1}{R_i} \]  \hspace{1cm} (5.21)

With the developed contact resistance models above, it is now possible to determine the contact resistance from the known contact penetration as shown in Figures 5.8 and 5.9. From (5.18) and (5.19) the contact radius of each asperity in contact with the beam can be found. Then, the equivalent resistance of the asperity is calculated by (5.20). Finally, the contact resistance of parallel asperities is found by (5.21). Figure 5.10 and 5.11 show the variation of contact resistance with actuation voltage for those cases. All parameters of the model are listed in the figures. Figure 5.10 shows that the asperity radius has strong influence on contact resistance especially at low actuation voltage. The reduction in contact resistance is initially very fast as the actuation voltage goes higher than the pull-in voltage. At very high actuation voltages, the resistance decreases at a slow rate and the discrepancy is small for variation of asperity radius. At 75V actuation voltage, the model predicted contact resistances are 0.32, 0.21, and 0.14\( \Omega \) for \( R=0.05 \), 0.1 and 0.2\( \mu \text{m} \) respectively.
5.4 Simulation and experiment results

Figure 5.10: Simulation of contact resistance of Ru-Ru contact under actuation voltages at 3 asperity radii.

Figure 5.11 shows that the standard deviation has low influence on the trend of contact resistance. Because the number of asperities is assumed to be identical, the transition gaps between asperity heights are larger for the higher standard deviation. In all cases the highest asperity has the same radius and therefore at initial contact, the contact resistances are identical. It starts to deviate when the beam interferes to the next asperities.

Figure 5.12 shows the comparison between the developed contact model (with $R=0.05\mu m$) and the measured switch resistances. The switch resistances from Figure 5.6 are re-plotted in Figure 5.12. The contact resistance model only predicts the contact resistance value while the measured resistance is the whole switch system resistance, which includes contact, beam, pads and wires. Therefore the contact model value is smaller than the measured switch resistance. During actuation with different voltage applied, only contact resistance changes. The resistance of all other parts (beam, pads, wires) are constant. The difference between measured switch resistance and the contact resistance model at 75V is around 1Ω Therefore it can be assumed that the constant is 1Ω, the shifted up contact model by 1Ω is also plotted on the Figure 5.12. The trend of the shifted contact model agrees well with the
5.4 Simulation and experiment results

Figure 5.11: Simulation of contact resistance of Ru-Ru contact under actuation voltages at 3 standard deviations.

measured data therefore the developed model predicts well the variation of contact resistance under actuation voltage. There is a substantial variation in the contact resistance, approximately between 140mΩ and 9.5Ω from 75 to 46V voltage applied.

Figure 5.13 shows the comparison between model and the measured switch resistances of 4 switches at cycle 9th. It shows that even the pull-in voltages of switches are different but the trends are quite similar to each other. And the contact model of 24 asperities with identical radius \( R = 0.05\mu m \) and Gaussian heights distribution predicts with high accuracy the Ru-Ru contact in the switch used in this test. The small discrepancy between the model and the measured results might be due to the difference between the distribution of the asperity heights of the real contact surfaces or the asperities difference.

In practice, the switch resistances (or contact resistances) are observed to respond differently under loading and unloading conditions. Figure 5.14 shows the measured switch resistances for several full cycles. If only a single cycle (e.g. cycle 9th)
5.4 Simulation and experiment results

Figure 5.12: Comparison between contact resistance model (with $R=0.05\mu m$) and the measured switch resistances at three cycles. The shifted contact model line shown on the figure is the addition of contact model with a constant value of the switch resistance other than contact.

is observed, this difference might be attributed to permanent deformation of the contact which would result in a smaller unloading resistance than the associated loading resistance. However, the resistances for subsequent cycles are quite similar to the prior cycle profiles.

The phenomenon of loading and unloading difference are observed in classical contact mechanics [118]. However, in this MEMS switch device, the presence of electrostatic forces might exaggerate the difference. The measurements indicate that the difference due to loading and unloading might mainly due to the overdrive effect which is higher at any given voltage pass $V_{pi}$ as the actuation voltage decreases and the nonlinear adhesion phenomenon but not due to the plastic deformation. It opens to the new research and development of the above model which can capture the unloading behaviour of MEMS switches.
5.4 Simulation and experiment results

![Image of Cycle 9th comparison](image1)

Figure 5.13: Comparison between contact resistance model (with $R=0.05\mu m$) and the measured switch resistances of 4 switches at cycles 9th.

![Image of Switch resistances](image2)

Figure 5.14: Switch resistances for full cycles: Loading (voltage increases) and Unloading (voltage decreases).

The developed contact model captures accurately the loading behaviour of the contact switch. The model can be used as a design tool to evaluate the performance
Table 5.2: Physical model parameters of multi-asperities ruthenium contacts

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol (unit)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus</td>
<td>$E$ (GPa)</td>
<td>414</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>$\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td>Surface energy</td>
<td>$\gamma$ (Jm$^{-2}$)</td>
<td>4.2</td>
</tr>
<tr>
<td>Asperity radius</td>
<td>$R$ ($\mu m$)</td>
<td>0.05</td>
</tr>
<tr>
<td>Asperity density</td>
<td>$D$ ($\mu m^2$)</td>
<td>20</td>
</tr>
<tr>
<td>Nominal contact area</td>
<td>$A_n$ ($\mu m^2$)</td>
<td>1.2</td>
</tr>
<tr>
<td>Standard deviation of asperity heights</td>
<td>$\sigma$ (nm)</td>
<td>4</td>
</tr>
</tbody>
</table>

and to improve the reliability of the micro-switch. The evaluation of dynamic performance will be presented in the next section.

5.4.2 Closing dynamics

Based on the previous contact resistance model development, several necessary parameters for multi-asperity nonlinear contact with adhesion used in this dynamic simulation are listed in Table 5.2.

Integrated the multi-asperities model as an boundary condition when the tip deflection reaches to the pad. The full dynamic model is developed. The dynamic behaviour of the model is evaluated in this section.

The model results of tip velocities at two actuation voltages (50 and 65V) are shown Figure 5.15. It is clearly shown that the higher the actuation voltage the faster the switch will initially close. Higher voltage also creates higher approaching speed to the contact. At 65V, the tip velocity at initial contact is 0.48m/s while it is only 0.26m/s at 50V. The contact speed is strongly related to the contact deformation therefore it is very important to the contact degradation effects. The tip velocity trends in Figure 5.15 show that the damping acts like a spring back force to reduce the beam velocity as it approaches to the substrate. This is due to the narrow gap between the beam and the substrate. The beam-to-gate gap is much smaller than the lateral dimension of the switch. It suggests that, under the high speed of a switch closure there is little time for the gas to escape and the flow
5.4 Simulation and experiment results

Figure 5.15: Tip velocity of the switch at actuation voltage of 50 and 65V

becomes more viscous. The presence of this viscous damping spring helps to reduce the speed at closing.

Figure 5.16 shows the simulated tip displacements of the micro-switch as a function of time at three different step actuation voltages: 50, 60 and 65V. The graphs show that the initial closing time decreases as the actuation voltage increases. The corresponding initial closing times are 1.40, 1.14, and 1.03\(\mu\)s at 50, 60 and 65V respectively. After initial contact, the switch bounces several times before settling. It is observed that the actuation voltage level affects the bouncing behaviour: settling time, number and amplitude of bounces, etc. As the actuation voltage increases, the number and amplitude of bounces decreases. There are about six bounces for 50V applied voltage whereas for 65V, there are only three bounces. Simulation also shows the settling time versus the actuation voltage trend. It decreases from 17.5\(\mu\)s at 50V to 14\(\mu\)s at 60V and around 6.2\(\mu\)s at 65V.

A setup as shown in Figure 5.17 was used to measure the voltage across the switch. The closed and open status of the switch is shown on the waveform recorded from the oscilloscope. When the switch is open, the switch voltage is equal to the
5.4 Simulation and experiment results

Figure 5.16: Tip displacements of the switch at actuation voltage of 50, 60, and 65V

Figure 5.17: Experimental setup for measuring the voltage across the switch as a function of time.

power supply voltage of 500mV. Otherwise, the voltage is lower than 500mV based on the contact penetration of the beam into the drain. The measurement results are shown in Figure 5.18. The switch bounces with a single step voltage actuation, and
5.4 Simulation and experiment results

the number of bounces decreases with increasing magnitude of the actuation voltage. It can be observed that there is an excellent agreement between the measured and the simulation data with respect to capturing the bouncing effect.

The number of bounces follows the same trend as in the simulation which is reduced from seven times at 50V down to just three times at 65V before settling. The initial closing times are around 1.5, 1.1, and 1\(\mu s\) at 50, 60, and 65V respectively. After going through several discontinuities, the beam makes permanent contact with the drain to the settling period at about 17.1, 12.9 and 6.4\(\mu s\) for the three applied voltages.

Figure 5.19 shows a comparison between the numerical results (Figure 5.16) and measured data (Figure 5.18) of the switch at tip-drain contact of initial closure \(T_{c1}\), the beginning of the first bounce \(T_{c2}\) and the beginning of the second bounce \(T_{c3}\). The simulation results show an excellent agreement with the experiment for 65V actuation voltage. The agreement for the initial closure is better than for the
5.4 Simulation and experiment results

Figure 5.19: Close times \( (T_{c1}, T_{c2}, T_{c3}) \) versus actuation voltage for simulation (lines) and experiment (scattered dots).

next bounce \( T_{c2} \) which in turn better than for \( T_{c3} \). The discrepancies call for further improvement in the dynamic model of the switch and/or the contact to acquire a better prediction of the bouncing behaviour.

The simulation results of the initial closing time, the number and duration of bounces, the settling time and the elimination of bounces are in excellent agreement with the experimental results. The consistency between simulation and experiment proves that the developed model with multi-asperity nonlinear contact can be used as a design tool to improve the performance and to evaluate the reliability of the micro-switch.
5.4 Simulation and experiment results

5.4.3 Model-based analysis of switch degradation effects during lifetime testing

This section reports on the transient analysis of the observed decrease in the actuation voltage of MEMS ohmic switches, under a stress condition. Figure 5.20 shows a SEM picture of a MEMS ohmic switch used in this work. The metal cantilever beam is fabricated from 8μm thick electroplated gold. The gate actuation electrode is positioned beneath the main beam area. The beam to gate gap and tip-gap, illustrated in Figure 5.5(b), are 0.6μm and 0.3μm respectively. The beam is 100μm long and 60μm wide. The fabrication process of the device has been described in detail in [109].

Unlike cycling testing which has been reported in many works [72][135], a hold-down test is designed to test the ability of the switch to withstand long term actuation. This is a key reliability factor for MEMS switches. A test was carried out in 26 days. The actuation voltage was measured at set intervals to observe the actuation voltage through-out the hold-down sequence. The measured actuation voltages were observed to decrease with time. Similar observations were reported in [82]. Figure 5.21 shows the result data on a set of five switches.

As seen in Figure 5.21, the actuation voltage dropped for all switches. Moreover, it was observed that the decrease was larger for those devices with a higher initial
actuation voltage. A decreasing actuation voltage eventually leads to the stuck-down failure of the devices. For the cantilever beam, two causes can account for this effect: (i) open state gap reduction due to mechanical (plastic) deformation, or (ii) beam stiffness reduction due to fatigue. To evaluate the role of these factors on the observed pull-in voltage decrease, a modification of the 2-D FDM as described above was used to evaluate the contribution of each factor.

The beam stiffness is a function of its shape and material properties. The shape is believed to be unchanged during the hold-down test. For that reason, the effect of the reduction of the material Young’s modulus is analysed to evaluate the contribution of stiffness to the actuation voltage changes.

When mechanical deformation occurs, the open state gap is reduced. The beam is tilted, as the deformation mainly occurs near the anchor [72][82]. At the equilibrium position, the effective initial tip-gap is shifted as: \( d_{\text{eff}} = d + \Delta d \). With \( \Delta d \) is positive for the upward curl and negative for the downward (Figure 5.22). The required actuation force to pull the beam to the closed position is changed accordingly.
It is difficult to include this phenomenon into equation (5.1) therefore an alternative technique is employed. The slope between the beam and the ground plane is created by tilting the plane of the ground instead of the beam. Figure 5.23 shows the approach adopted to simulate the changing of the open state gap.
As the open state gap is reduced, the initial condition of the model is modified. The tip-gap ($tg$) is also reduced accordingly. The initial distance $g$ in equation (5.3) is different from point to point of finite subintervals along the beam above the Gate area.

A simulation code for equation (5.1) is implemented in MATLAB. Two degradation mechanisms, stiffness change and tip-gap change due to plastic deformation, are taken into account. The static actuation voltage is found by gradually increasing the applied voltage until the tip reaches the drain. Figure 5.24 presents the results of the FDM analysis of the static pull-in voltage as a function of the two degradation mechanisms. It shows that the pull-in voltage depends more on tip-gap than on beam stiffness.

Tip-gap measurements were carried out during the test. The tip to substrate gap was measured using a Zygo White Light Interferometer at the open state (no voltage
5.4 Simulation and experiment results

Figure 5.25: Comparison between measured data with simulation analysis on two degradation mechanisms.

applied) and at the closed state. The experimental tip-gap is the difference between the two measurements. A strong correlation was found between actuation voltage drift and the measured tip-gap. Figure 5.25 shows the experimental data and the comparison with a simulation of actuation voltage versus tip-gap and stiffness.

Under actuation, stress is induced in the beam. If these stresses exceed the yield stresses of the material, permanent deformation can occur. Over time, these deformations reduce the tip-gap and opening force of the beam, leading to reduced performance or eventual failure to open. This is particularly a problem with MEMS structures manufactured on soft metals such as copper or gold where plastic deformation is generally the first material failure to occur in the beam [82]. It is evident that under hold-down testing, the tip-gap is reduced resulting in the observed decrease of actuation voltage. It agrees well with plastic deformation near anchor points seen using SEM analysis as reported in [72].

There is a good agreement for the data in the case of the theoretical Young's
5.4 Simulation and experiment results

modulus which demonstrates that the pull-in voltage decrease is mainly due to the mechanical deformation. The contribution of stiffness is small and for lifetime prediction it is assumed to be unchanged during the test.

Figure 5.26 shows the comparison of a 2-D simulation, a 1-D lumped element model (as used in [137]) and experimental data on actuation voltage versus tip-gap. If the evaluation is merely based on this 1-D lumped model which does not account for the real shape of the beam, the conclusion might suggest that there is a reduction of stiffness as well tip-gap during hold-down testing. The presented 2-D model which takes account of the real shape of the beam is believed to achieve higher accuracy with experimental observations and indicates that mechanical deformation is the dominant contributor to the actuation voltage change.

Based on this observation, the tip-gap change can be determined from the measured actuation voltage decrease shown in Figure 5.21 using the simulation result. A curve based on theoretical Young's modulus ($E=78$ GPa) in Figure 5.25 is used.
5.4 Simulation and experiment results

in this interpolation. The interpolated tip-gap is drawn in Figure 5.27. It shows that the switch with larger initial gap exhibits a bigger gap change.

Converting the reduced tip-gap trend in Figure 5.27 into tip-gap change data, another graph as shown in Figure 5.28 is obtained. In log-log scale, a strong linearity trend of tip-gap change over time is observed. This correlation indicates that a model could be developed to predict these behaviour, for both stress dependence and time.

The standard creep equation relates stress, time and other factors such as temperature and material properties was given by \[145][82]\n
\[
\varepsilon = \text{creep strain} = A_0 \sigma^m t^n \exp \left( \frac{-Q}{RT} \right)
\]

where \(A_0\) is a fitted gain constant, \(\sigma^m\) and \(t^n\) are stress and time variables, where \(m\) and \(n\) are constant. \(Q\), \(R\) and \(T\) represent activation energy, Boltzmann’s constant and absolute temperature. As temperature is constant during testing, the temperature exponent term is constant.
5.4 Simulation and experiment results

Figure 5.28: Converted tip-gap versus time (log-log scale).

Table 5.3 shows the actuation voltage and equivalent peak stress applied on each tested switch. The peak stress, which is tensile force over the anchor area, was extracted by using Finite Element Analysis [82].

Fitting equation (5.22) to the converted data in Figure 5.28 and peak stress values in Table 5.3, equation (5.22) for this study now becomes

\[ \Delta gap = A_1 \sigma^m t^n \]  

(5.23)

<table>
<thead>
<tr>
<th>Switch No.</th>
<th>Actuation [V]</th>
<th>Peak stress, ( \sigma ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40.7</td>
<td>26.3</td>
</tr>
<tr>
<td>2</td>
<td>45.6</td>
<td>31.8</td>
</tr>
<tr>
<td>3</td>
<td>49.3</td>
<td>36.18</td>
</tr>
<tr>
<td>4</td>
<td>51.1</td>
<td>38.42</td>
</tr>
<tr>
<td>5</td>
<td>47.2</td>
<td>33.7</td>
</tr>
</tbody>
</table>
5.4 Simulation and experiment results

Figure 5.29: Converted Tip-gap versus time and the fitted model (log-log scale).

with $m = 4.01, n = 0.202, A_1 = 2.7 \times 10^{-39}$

The high stress dependence, $m=4$, suggests that small changes in the stress, will result in large reductions in the gap. The gap decrease with time coefficient is $n=0.202$.

Applying equation (5.23) with the peak stress values from Table 5.3, measured and simulated tip-gap values versus time are shown in Figure 5.29. It shows a good fit between the model and measured data. These strong correlations with early data suggest the model can be used to extrapolate the long-term behaviour of the switch.

It is shown that the lower the peak stress, the higher the expected lifetime. The developed model (5.23) can be used in the inverse way to estimate the maximum peak stress acceptable to meet the expected lifetime criterion. For high reliability ohmic switches, it is anticipated that a maximum tip-gap change of 10% of the initial value can occur in 10 years. For the switch considered in this work, this implies a tip-gap change of 30 nm.
A horizontal line in Figure 5.29 shows this criterion. It shows that all the tested switches do not meet the criterion. At 26.3 MPa peak stress, the tip-gap change exceeds the criterion after just 30 days. However, if the peak stress is reduced to 20 MPa, the line model falls within the expected lifetime criterion. The model predicts that for less than 10% tip-gap change after 10 years, the peak stress applied on this particular MEMS cantilever switch must be less than 20 MPa.

It is seen that by reducing the peak stress by 24% (from 26.3 to 20 MPa) a huge improvement can be achieved in the lifetime expectation of the device (from 30 days to 10 years).

5.5 Summary

In this chapter, the dynamic behaviour of an ohmic-contact electrostatic MEMS switch, including bouncing behaviour, has been modeled. The proposed model takes into account the real switch geometry, squeeze-film damping effect and nonlinear elastic-plastic contact with adhesion. A high accuracy multi-asperity nonlinear contact with adhesion model was developed to simulate the complex interaction during closure of the contact on the drain. The model was used to simulate the deformation of the contact, the contact resistance, switching speed, the tip displacement and the bouncing behaviour of the switch. The simulation results of the contact resistance, initial closing time, the number and duration of bounces and the settling time are in excellent agreement with experimental results.

The analysis on multi-asperity contact model showed that the contact roughness can be modeled as a number of asperities with identical radius and the asperities' heights follow Gaussian distribution law. The evaluation between asperities’ radius and standard deviation on the contact resistance has been carried out. It showed that the asperities’ radius have more influence on the initial contact resistance than the deviation between asperities.

The FDM approach presented in this work was used to analyse the effect of different device geometries on lifetime and reliability which could not be done on
the basis of an empirical fit or analyses based on existing analytical models. The proposed method is less time consuming than a full 3-D model possible with Finite Element Method or Molecular Dynamics. The work demonstrates that the mechanical (plastic) deformation of the switch is the dominant factor in the actuation voltage degradation.

The relation between degradation mechanisms and peak stress condition has been identified. A fitted model is developed based on the measured data to predict the lifetime of a particular gold cantilever beam design. It shows that the peak stress is an important factor in the reliability of the switch in long-term actuation.

The consistency between simulation and experiment prove that the developed model with multi-asperity nonlinear contact can be used as a design tool to improve the performance and to evaluate the reliability of the microswitch. The small discrepancies call for an improved model that comprises the elasto-plastic contact mechanics and process variance should be developed in the future works.
Chapter 6

Energy-based approach to adaptive pulse shaping control

*Everything should be made as simple as possible, but no simpler*

*Albert Einstein*, physicist (1879-1955)

This chapter presents a closed-form analysis to design a pre-shaped open-loop driving actuation waveform to reduce the bouncing effect while maintaining fast switching of a MEMS contact switch. A single-degree-of-freedom model and the principle of energy conservation were utilized to design a shaped voltage waveform to close the switch with low impact speed. The method can easily adapt the voltage waveform to the variance of pull-in voltages due to imperfect manufacturing and to observed pull-in voltage drift during operation. The analytical calculations of the actuation pulse and the closure time of the switch with near zero velocity are compared versus simulation results and are validated experimentally on a set of cantilever switches. It is further shown that the analysis is also applicable for electrostatically actuated devices with more general geometries.
6.1 Introduction

As discussed in Chapter 3, electrostatically actuated MEMS devices exhibit a dynamic instability point, referred to as the pull-in instability, which results in rapid, unstable collapse of the switch to the substrate affecting the lifetime of the system [134][3]. Furthermore, due to high impact forces on closure, the switch bounces backward several times, interrupting the output signal thus increasing the switching time. To achieve the required performance for commercial application, control techniques can be used to improve the MEMS device performance characteristics.

From a control point of view, MEMS devices can be controlled in open-loop and closed-loop systems [146]. Both approaches have their relative advantages and disadvantages, nonetheless both are challenging since the time responses involved in MEMS devices are often several orders of magnitude faster than of macro-scale devices [147]. In the open-loop control approach, the input driving waveform is tailored based on the specifications of the device and the required dynamic performance. This method requires fewer additional circuits than the closed-loop method but suffers from low robustness to variation of the parameters of the system due to fabrication inconsistencies and lifetime drift. On the other hand, the feedback control approach [148][149] is significantly less sensitive to changes in system parameters and so produces a better dynamic performance output. The practical realization of a feedback control for MEMS is problematic since it requires additional sensing and control elements to be integrated with the MEMS device on the integrated circuit chip. In terms of cost and complexity with limited space and power consumption in industrial applications, the open-loop (pre-shaping) method is preferable and is the subject of this work.

The open-loop input shaping technique [150][151] which is based on the derived parameters of the system has been widely and successfully used in controlling linear or linearized systems. The method, which is also called "Posicast control", has been deployed in MEMS systems to remove the residual vibration [152]. However this method is only applicable to types of linear actuation MEMS devices, such as comb drives. For a DC-contact switch and other MEMS applications with nonlinear actuation, this method is no longer applicable. The possibility of using soft-landing
6.1 Introduction

Figure 6.1: Schematic of input shaping and system response for nonlinear actuation DC-contact switch.

Waveforms to actuate nonlinear MEMS devices to enhance performance has been reported in various works [153][154][107]. The effectiveness of the method was proven, but the methods used a complex analytical-numerical solution and not a simple analytical one as presented here.

An open-loop control approach which is based on energy conservation for electrostatically actuated MEMS has been independently developed by Czaplewski et al. [90] and Chen et al. [155]. The method comprises two pulses as shown in Figure 6.1. The actuation pulse has a high voltage but short duration which provides the necessary amount of energy to actuate the structure to the contact with near-zero velocity. The hold voltage is then applied upon contact closure to keep the switch from opening. The method has been demonstrated in both simulations and experiments in recent papers [66][156]. However, from an engineering perspective, it is difficult to apply the analytical-numerical approach, which is used in [107][90][156], or semi-analytical approach with implicit elliptical integral, which is used in [155], to an industrial application. In other words, an explicit solution would provide a better basis for the application of the control system in a commercial MEMS switch.

Guo et al. [66] introduced a comprehensive solution to calculate the timing of the pulses, however, the method is based on the assumption of a fixed actuation force. To the best of our knowledge, a closed-form solution for voltage levels and timing of the pulses has not been introduced in the nonlinear MEMS field yet. In this work I propose a closed-form solution for waveform parameter definition which may make
6.2 MEMS switch contact bounce

the pulse shaping control method sufficiently simple for deployment in a commercial application environment.

The open-loop method is sensitive to the variation of the switch [90][66]. To accommodate this system variation, Blecke et al. [157] developed a simple learning control algorithm whereas Ou et al. [158] employed an online tuning scheme to vary parameters of the above mentioned input shape. Both methods are efficient for handling uncertainty and damping. However, basically these are closed-loop control methods; it either needs a smart chip to run the algorithm or requires several manual steps to adapt the waveform, making it unattractive for real world application.

In this chapter, a closed-form analytical energy-based method is presented to accurately shape the input to eliminate bouncing while maintaining fast switching for the device. Furthermore, in order to better control parameter variation, an adaptive control which is based on measured pull-in voltage to modify the timing and amplitudes of pulses is developed in this work. Experimental measurement on a single contact cantilever MEMS switch has been carried out to evaluate the developed methods.

The remainder of the chapter is organized as follows: Section 6.2 describes the dynamic operation of the switch with bouncing under normal step control, Section 6.3 presents an one-dimensional model and mathematical development of the input shaping technique. An adaptive control method to cater for process uncertainty and operational drift is presented in Section 6.4. Finally, experimental comparisons are presented in Section 6.5.

6.2 MEMS switch contact bounce

The electrostatically actuated ohmic contact RF-MEMS switch as shown in Figure 5.5(a) is considered. It is shown again here in Figure 6.2 for more convenience. The cantilever beam is fixed at one end and has one contact point at the other end. To reduce the threshold voltage, the beam near the fixed end is designed with a smaller cross-section area than the driven area. The electrostatic force to close the switch
Figure 6.2: Top-view, SEM micrograph of the MEMS switch.

is generated by a voltage applied between the beam and the area on the substrate directly beneath the wide rectangular region. The switch is made of gold.

When a sufficiently large voltage is applied to the actuation electrodes, the resultant electrostatic force pulls the beam toward the contact pad. With a step actuation voltage, the continuous applied electrostatic force makes the beam approach the contact at a very high speed. This results in high impact forces between the tip and the contact pad. Consequently, the switch rebounds several times before making permanent contact [88][66].

The measurement setup shown in Figure 6.3 is used to observe the signal across the switch during the switching operation. Figure 6.4 shows the 500mV DC signals across the switch at two simple step actuation voltages, 50 and 60V. It shows that the bouncing phenomenon increases the settling time of the switch. At 50V, the initial contact occurs at around $1.5\mu s$, but the switch bounces 7 times before completely closing at around $17\mu s$. Due to the bouncing problem, the signal is interrupted and the closing time is extended significantly.

Another side effect of the large impact velocity is the local deformation at the contact points. The deformation level can be observed as the voltage level at the output. The higher the deformation at the contact points, the lower the on-resistance between two contacts, thus giving lower voltage drop across the switch. The switch voltages in Figure 6.4 show that the higher the voltage applied, the higher the deformation created, especially at the initial contact. This deformation is believed to be one of the main contributors to the mechanical failure of the switch. In
6.3 Energy-based control

6.3.1 One-dimensional model

Figure 6.3 shows a schematic and a lumped element model of the switch in Figure 6.2. The detail was described in Chapter 5. The mass-spring-damper system as shown in Figure 6.5(b) was created as a single-degree-of-freedom (SDOF) model to approximate the actuation motion of the tip of the beam. At the equilibrium position, the mass to substrate distance is the initial distance between the tip and the drain pad, $d$. The force exerted, $F_e$, on the mass is the electrostatic force created
6.3 Energy-based control

Figure 6.4: Experiment setup for measuring signal running across the switch.

Figure 6.5: (a) 3-D angle-view schematic of the switch. (b) Spring-mass-damper model.

by beam-to-gate potential voltage as shown by the row of downward arrows in Figure 6.5(a).

As the tip is displaced a distance \( x \), the electrostatic force is the integral along the beam from \( l_1 \) to \( l_1 + l_2 \) as shown in Figure 6.6(a). The result of this integral calculation provides a good approximation of the real force. However, the high
6.3 Energy-based control

Figure 6.6: Electrostatic force. (a) Integration analysis. (b) Simplified analysis.

complexity makes it difficult for future analysis. To simplify the calculation, the force is assumed to be concentrated in the middle of the gate, Figure 6.6(b). With the dimensions of the switch used in this work, this force is defined by

\[ F_e = \frac{\varepsilon_0 A_g V^2}{2(g - x/2)^2} \]  

(6.1)

where \( \varepsilon_0 \) is the permittivity of air, \( A_g \) is the area of the energized beam which is equal to the gate area, \( V \) and \( g \) are the potential voltage and the initial gap between beam and gate respectively, \( x \) is the instantaneous deflection of the mass (or the tip) toward the substrate.

At the closed position, the error between the simplified and the integral analyses is at a maximum value, and is 3.6% for this switch. For this wide beam switch, the additional force due to fringing field is less than 1% [93], therefore it is neglected. With this evaluation, the simplified approximation of (6.1) is acceptable and it is used for future analysis.

The stiffness, \( k_b \), and natural frequency of the beam were obtained by Finite Element Analysis (FEA) of a full three-dimension (3-D) model of the switch. The commercial FEA software package ANSYS was used for this purpose. The displacement of the cantilever is small (maximum at 0.26\( \mu \)m) compared to the length, so \( k_b \) is assumed to be linear, of 320N/m. The static pull-in voltage of the model is 41.6V. The effective mass, \( m \), is extracted from the stiffness and natural frequency analysis based on harmonic oscillation theory.

The beam-to-gate gap is much smaller than the lateral dimension of the switch.
6.3 Energy-based control

It suggests that, under the high speed of a switch closure there is little time for the gas to escape and the flow becomes more viscous. The motion of squeezed-film damping with compressible gas between two movable plates is governed by Reynolds lubrication equation [66][125].

Darling et al. [125] used a Green function method to solve Reynolds equation for various boundary conditions. The method can be extended to derive the squeezed damping forces (damping and elastic force) for a cantilever by assuming one edge is closed (no venting) and the beam has a parabolic deflection. Adopting that method to this switch, the normalized squeezed air forces versus mechanical frequency can be found as shown in Figure 6.7. It shows that the cut-off frequency is 481kHz where gas compression starts to be dominant over viscous energy dissipation. Estimation using Simulink provides that at 45-70V actuation voltage the switch oscillates at around 450 to 600kHz, while it oscillates at a natural frequency of 200kHz when the voltage is removed. As the mechanical frequency of the beam varies with actuation voltages, the damping coefficient and viscous damping spring are not constant. For this reason, the values used in the simulation are estimated based on the prevailing operating conditions.

Furthermore, the preliminary estimation from (6.1) and Figure 6.7 provide that the maximum percentages of squeezed-film damping forces over the electrostatic force range from 6% at 70V to 14% at 45V voltage applied. At high actuation voltages, the damping is neglected to simplify calculation. The equation describing the lumped system dynamics under electrostatic force is expressed as

$$m\ddot{x} + c_d\dot{x} + (k_b + k_d)x = -\frac{\varepsilon_0 A_y V^2}{2(g - x/2)^2}$$  \hspace{1cm} (6.2)

where $k_b, k_d, m$ are beam stiffness, squeezed air damping stiffness and effective mass respectively and $c_d$ is the damping factor of the system.

Several necessary parameters for the model are listed in Table 6.1
Figure 6.7: Maximum squeezed-film (Elastic) and viscous damping (Dissipative) force due to air around the cantilever beam as a function of oscillation frequency.

### 6.3 Energy-based control

#### 6.3.2 Pulse shaping mathematical development

The method of pre-shaping the actuation voltage waveform to minimize the impact force and eliminate the bouncing effect to improve the operation and reliability of the switch has been widely reported [107][90][155]. The idea of a dual-pulse voltage waveform to achieve a "soft landing" and "no bouncing" of the switch is shown in Figure 6.8. $V_a$ is the applied voltage, which must exceed the pull-in voltage of the switch. For this applied voltage, the pulse width, $t_a$ is calculated to create enough momentum to actuate the switch to closure with zero velocity at the substrate after the time $t_c$. At this moment, the holding voltage $V_h$ is applied to keep the switch from restoring back to the open position. The force created by $V_h$ should be higher than the pull-back force of the spring, thus $V_h$ must satisfy

$$V_h \geq (g - d/2) \sqrt{\frac{2k_e d}{\varepsilon_0 A_g}}$$
6.3 Energy-based control

Table 6.1: Physical model parameters of one-dimensional model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol (unit)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permittivity of air</td>
<td>$\varepsilon_0 (F/m)$</td>
<td>$8.85 \times 10^{-12}$</td>
</tr>
<tr>
<td>Gate area</td>
<td>$A_g (\mu m^2)$</td>
<td>$2.4 \times 10^3$</td>
</tr>
<tr>
<td>Beam-to-gate gap</td>
<td>$g (\mu m)$</td>
<td>0.60</td>
</tr>
<tr>
<td>Tip-gap (mass travel)</td>
<td>$d (\mu m)$</td>
<td>0.26</td>
</tr>
<tr>
<td>Beam spring-constant</td>
<td>$k_b (N/m)$</td>
<td>320</td>
</tr>
<tr>
<td>Effective mass</td>
<td>$m (kg)$</td>
<td>$2 \times 10^{-10}$</td>
</tr>
</tbody>
</table>

6.3.2.1 Ideal input waveform

The purpose of the mathematical development is to achieve near-zero velocity of the switch at the point $t_c$. As observed in Figure 6.8, the applied voltage is 0V from time $t_a$ to $t_c$. The equivalent SDOF dynamics, with negligible damping, from 0 to $t_c$ can be rewritten as

$$m\ddot{x} + kx = \begin{cases} \frac{a}{2(g - x/2)^2}, & \text{for } t \leq t_a \\ 0, & \text{for } t_a < t \leq t_c \end{cases}$$  \hspace{1cm} (6.3)

where $a = \varepsilon_0 A_g V_a^2$, $k$ is the effective spring constant of the system.

The switch accelerates and moves toward the drain when a constant voltage $V_a$ is applied. At $t_a$, the electrostatic force ceases and the switch continues to move downward due to its momentum. From $t_a$ to $t_c$, the kinetic energy of the mass is gradually transferred to potential energy in the spring. At time $t_c$, $x = d$, the mass velocity is expected to be zero, therefore the system energy is entirely potential energy. Based on the law of energy conservation, the work done on the system by the electrostatic force $F_e$ is the energy of the system at this closed position

$$\frac{kd^2}{2} = \int_{0}^{x_a} \frac{a}{2(g - x/2)^2} \, dx$$  \hspace{1cm} (6.4)
6.3 Energy-based control

Figure 6.8: Schematic of the ideal dual-pulse waveform input. $V_a$ is actuation voltage and is usually higher than the holding voltage $V_h$. $t_a$ denotes the width of the actuation voltage and $t_c$ is the closing time.

Solving (6.4), $x_a$ can be found as

$$x_a = 2 \left( g - \frac{a}{kd^2} + \frac{a}{2} + \frac{a}{g} \right)$$ \hspace{1cm} (6.5)

To determine equivalent time $t_a$ at position $x_a$, (6.3a) is considered. At any
6.3 Energy-based control

instantaneous position, \( x \), between 0 and \( x_a \) the work done by the electrostatic force on the switch over the distance \( x \) is defined as

\[
W = \int_0^x \frac{a}{2(g - \zeta/2)^2} \, d\zeta = -\frac{ax}{g(2g - x)} \tag{6.6}
\]

Based on the law of conservation of energy, the sum of kinetic and potential energies at the position \( x \) of the switch must equal to the work done by the electrostatic force on the switch from 0 to \( x \). Therefore, one obtains

\[
\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = W \tag{6.7}
\]

By replacing (6.6) into (6.7) noting that \( v = dx/dt \), (6.7) can be rewritten as

\[
\frac{1}{2}kx^2 + \frac{1}{2}m \left( \frac{dx}{dt} \right)^2 = -\frac{ax}{g(2g - x)} \tag{6.8}
\]

Solving (6.8) for \( dt \), it can be found as

\[
dt = \frac{1}{\sqrt{\frac{2}{m} \left( \frac{ax}{g(2g - x)} - \frac{1}{2}kx^2 \right)}} \, dx \tag{6.9}
\]

And time \( t_a \) at position \( x_a \) can be found by integrating both sides of (6.9)

\[
t_a = \int_0^{t_a} \, dt = \int_0^{x_a} \frac{1}{\sqrt{\frac{2}{m} \left( \frac{ax}{g(2g - x)} - \frac{1}{2}kx^2 \right)}} \, dx \tag{6.10}
\]

The integration in (6.10) can be solved by using elliptical integrals which is quite complex or numerically using mathematical tools (e.g. Matlab). It cannot be solved
6.3 Energy-based control

explicitly. To obtain an analytical solution, suited to adaptive control, (6.10) can be manipulated to achieve an approximate solution with a small error resulting from the approximation.

Looking back to (6.6) and (6.10), one can observe that the variable $x$ in the denominator of (6.6) makes it difficult to solve (6.10). To eliminate this variable in the denominator, (6.6) is expanded using a Taylor series approximation in a neighborhood of $x = 0$ to obtain

$$W = \frac{ax}{g(2g - x)} = \frac{ax}{2g^2} + \frac{ax^2}{4g^3} + ....$$

(6.11)

Replacing (6.11) into (6.7) and ignoring the higher order terms of the Taylor expansion, (6.10) now becomes

$$t_a = \int_0^{x_a} \frac{1}{\sqrt{\frac{2}{m} \left( \frac{ax}{2g^2} + \frac{ax^2}{4g^3} - \frac{1}{2}kx^2 \right)}} dx$$

(6.12)

The integral in (6.12) has a form of

$$t_a = \int_0^{x_a} \frac{1}{\sqrt{Ax^2 + Bx}} dx$$

(6.13)

with

$$A = \frac{a - 2kg^3}{2mg^3}, B = \frac{a}{mg^2}$$

Equation (6.13) can be solved explicitly to yield the closed-form solution of $t_a$. 

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6.3 Energy-based control

as a function of $x_a$ as

$$t_a = -\sin^{-1} \left( \frac{\sqrt{A|x_a|}}{B} \right) + \frac{\sin^{-1}(1)}{\sqrt{|A|}}$$ (6.14)

To find $t_c$, (6.3) is considered. Due to the input voltage ceasing at $t_a$, the system oscillates under free movement from $t_a$ to $t_c$. The switch reaches its maximum displacement with zero velocity at the substrate, hence the amplitude of the motion equation is $d$. The sinusoidal motion equation of (6.3) can be described as

$$x(t) = d \sin \left( \sqrt{\frac{k}{m}} t + \phi \right)$$ (6.15)

where $\phi$ is the phase of the system, which is determined by the initial condition: $t = 0, x(0) = x_a$. The phase $\phi$ can be found as

$$\phi = \sin^{-1} \left( \frac{x_a}{d} \right)$$ (6.16)

The holding time between $t_a$ and $t_c$ is given by: $t_h = t_c - t_a$. When the mass arrives at the substrate, it reaches the maximum displacement, given by $x(t_h) = d$, therefore

$$\sin \left( \sqrt{\frac{k}{m}} t_h + \sin^{-1} \left( \frac{x_a}{d} \right) \right) = 1$$ (6.17)

Following the trigonometric function, one obtains

$$\sqrt{\frac{k}{m}} t_h + \sin^{-1} \left( \frac{x_a}{d} \right) = \frac{\pi}{2}$$ (6.18)

The time that the switch needed to travel from $x_a$ to $d$ (substrate), $t_h$, can be
6.3 Energy-based control

The time delay and the total time can be explicitly calculated for any actuation voltage using (6.14) for $t_a$ and (6.19) for $t_c$.

In conclusion, time $t_a$ and $t_c$ can be explicitly calculated for any actuation voltage using (6.14) for $t_a$ and (6.19) for $t_c$.

To evaluate the developed analytical formulation, a Simulink model is constructed using (6.3) to simulate the behaviour of the switch. The explicit value is calculated for $t_a$ input to the Simulink model and modified as required to achieve near-zero velocity of the simulation system upon closing. This gives a measure of the error associated with the Taylor series approximation in the explicit solution. Figure 6.9 shows the comparison between analytical calculation with damping ne-
6.3 Energy-based control

Table 6.2: Pulse width comparison of calculation and simulation with damping

<table>
<thead>
<tr>
<th>$V_a$ (V)</th>
<th>Analytical, $t_a$ ($\mu$s)</th>
<th>Simulation, $t_a$ ($\mu$s)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>1.084</td>
<td>1.144</td>
<td>5.6</td>
</tr>
<tr>
<td>50</td>
<td>0.868</td>
<td>0.916</td>
<td>5.5</td>
</tr>
<tr>
<td>55</td>
<td>0.715</td>
<td>0.754</td>
<td>5.4</td>
</tr>
<tr>
<td>60</td>
<td>0.601</td>
<td>0.633</td>
<td>5.3</td>
</tr>
<tr>
<td>65</td>
<td>0.513</td>
<td>0.541</td>
<td>5.5</td>
</tr>
<tr>
<td>70</td>
<td>0.443</td>
<td>0.466</td>
<td>5.3</td>
</tr>
</tbody>
</table>

The excellent agreement between the analytical model and the Simulink simulation without damping (error is less than 0.27%) shows that the approximation using the Taylor series in (6.11) is sufficiently accurate. With a constant air damping ratio added into the Simulink model, higher errors are found between analytical and Simulink. Table 6.2 shows the pulse width errors between calculation and simulation with damping included at six actuation voltages. Higher errors between analytical and numerical simulation are observed for smaller input voltages, however, the errors are relatively fixed around 5.4%.

6.3.2.2 Practical waveform

In practice, the ideal waveform developed in the previous part cannot be produced. Intrinsic parameters, such as capacitance, inductance and resistance inside the signal generator, amplifier and the switch affect the speed of the signal, e.g. some amount of time is required for the electron charging or discharging of the specific capacitor. Figure 6.10 shows the shape of the ideal and practical actuation pulses.

In the previous part, the conservation of energy was used. The energy of the voltage signal is transferred to the system by the electrostatic force as shown in (6.1). The force is proportional to square of voltage. Here, again, the law of conservation of energy can be used to evaluate both signals. The energies of both ideal and
non-ideal waveforms are

\[
\int_0^{t_2} V_1^2 dt = \int_0^{t_{ap}+t_f} V_2^2 dt \tag{6.20}
\]

In most cases, the rise time, \(t_r\), and fall time, \(t_f\), are equal. Assuming a linear rise and fall profile, the practical actuation time \(t_{ap}\) can be found from (6.20) as

\[
t_{ap} = t_a + \frac{1}{3} t_r \tag{6.21}
\]

6.4 Adaptive control

6.4.1 Tip-gap versus pull-in voltage

The experimental data in [82] show that there is a large variation in the observed pull-in voltage of switches after manufacture and during operation. This variation is attributed to plastic deformation of the switch hinge area, where the highest stress is
6.4 Adaptive control

Figure 6.11: Schematic of switch with different tip-gaps.

concentrated. This results in the beam to drive electrode gap varying across switches from a single wafer and also varying over the operational lifetime of the switch. The paper reports a strong relation between pull-in voltage and tip-gap of the switch. The switch presented in [82] is slightly different in design and dimensions from the switch used in this work, however both were fabricated by the same process. Thus, a similar pull-in versus tip-gap relationship is expected to exist in the switch used in this work.

At the equilibrium position, the effective initial tip-gap is shifted as: \( d_{eff} = d + \Delta d \). With \( \Delta d \) is positive for the upward curl and negative for the downward (Figure 6.11). Consequently, the initial position of the mass to the substrate in Figure 6.5b is \( d_{eff} \). The required actuation force to pull the mass to the closed position is changed accordingly. The relation between the pull-in voltage and the tip-gap with
6.4 Adaptive control

The constant error is believed to be due to the design differences between the two switches. In fact, the spring-constant might slightly change due to deformation, however the similar trend between (6.22) and experiment shows that the pull-in variation is mainly due to inherent stress variation. The tip-gap variation is dominant over the spring-constant variation. Therefore, the change in spring-constant is neglected.

### 6.4.2 Proposed adaptive control

Based on the observed relation between tip-gap and pull-in voltage, a simple adaptive control method is proposed here for which a flowchart is shown in Figure 6.13.
6.5 Experiment evaluation

First, a static pull-in voltage is measured by gradually increasing the input while observing the continuity across the switch. Second, the effective tip-gap is calculated by (6.22) with the measured pull-in voltage. The effective beam-to-gate gap is found as, $g_{\text{eff}} = g + \Delta d/2$, $\Delta d$ is the difference between calculated and expected tip-gap. Third, the energy-based control method developed in previous part is utilized. The parameters of input shaping are explicitly calculated using the effective gap using equations (6.5), (6.14), (6.19) and (6.21). $N$ is a set number of cycles for every loop of reshaping. The duration between two reshaping runs ($N$) is determined by rate of change versus number of cycles. The proposed method can be applied to other cases, if the spring-constant and/or tip-gap variations are known in relation to the pull-in voltage.

6.5 Experiment evaluation

The measurement setup shown in Figure 6.3 was used to evaluate the developed methods. The rise and fall times of this measurement setup are $t_r = t_f \approx 0.2\mu s$. The measurement method is limited as the observed signal across the switch could not give the exact displacement with time. However, it provides accurately the impact time, impact level and the bouncing behaviour of the switch. Furthermore, unlike the displacement dynamics measurements using laser Doppler vibrometry system [107][90], this setup does not require removing the zero level encapsulation of the switch and therefore the test condition is as in normal switch operation.

The calculations are based on the SDOF model derived for an ideal switch and therefore it is not a perfect match to the real fabricated switch. Hence, the calculated parameters of the input waveform have to be tested and refined with experiment to achieve near-zero contact velocity with bouncing eliminated.

A set of five actuation voltages of 45, 50, 55, 60 and 65V were chosen to experimentally evaluate the developed theory. Using (6.5) and (6.14), the applied pulse width, $t_a$, is computed for each actuation voltage based on the data in Table 6.1. The actuation pulse signal as shown in Figure 6.10 was applied to actuate the switch. By observing the signal output, especially the initial contact, of the switch, the impact
6.5 Experiment evaluation

Figure 6.13: Flow chart of the proposed adaptive control.
6.5 Experiment evaluation

Figure 6.14: Switch voltages output at five one-pulse actuation voltages of 45, 50, 55, 60 and 65 on a 41V pull-in switch. Arrows point out that the beam closed to the drain at a very low speed before restoring back.

level can be evaluated. If the closing speed of the beam is high a low voltage is observed at the switch voltage output as contact deformation is significant due to the high impact force. When the beam contacts the drain at a near-zero velocity, a slightly reduced voltage peak is observed in the switch voltage output. The lowest observed peaks are recorded to compare with $t_c$ calculated using (6.19).

As mentioned above, a small change in the calculated pulse duration is expected to be required for the calculated one-pulse actuation voltage to work experimentally. Figure 6.14 shows the measured switch voltages at five actuation voltage of 45, 50, 55, 60 and 65V on a 41V pull-in switch. In each case, the first (upward) and second (downward) peaks are created by the coupling effect due to the rising and falling edges of the one-pulse actuation voltage. This phenomenon was explained briefly in [66]. The lower the actuation voltage, the longer the duration of the pulse required to supply enough energy to close the beam. When the duration of the pulse is
6.5 Experiment evaluation

long enough the beam is closed in contact with the drain before restoring back, the third peak (arrow pointed) was observed at the output. As explained previously, the shorter the peak observed, the lower the speed of the beam when it contacts the drain. The small third peaks in Figure 6.14 show that the switch was closed with the near-zero velocity. The one-pulse was applied, there was no second pulse to keep the beam in the closed position so the beam is released immediately after touching the drain.

Figure 6.15 shows a comparison between the applied experimental pulse widths and measured contact times for near-zero velocity impact and values calculated using the energy-based control method, with and without squeezed-film air forces, for a zero velocity impact. The values of $t_{AP}$ and $t_c$ are derived from (6.14), (6.19) and (6.21) with $\tau = 0.2\mu s$. The calculated results show a good agreement with the experiment. In the case with squeezed air forces neglected, the higher the actuation voltage the better the accuracy of $t_{AP}$ between calculation and experiment. The errors are 12.2% for all five cases compared with experiment. The error trend is in excellent agreement with the preliminary prediction in Section 6.3.

The estimated viscous and squeeze-film damping forces for the switch as a function of frequency were shown in Figure 6.7. During the period from 0 to $t_c$, these forces combine with the spring force of the beam to oppose the electrostatic attraction force. Using the data from Figure 6.7 the combined damping forces in the time from 0 to $t_a$, where the oscillation frequency is 450-600kHz (45-70V voltage applied), can be estimated at 14$\mu N$. This is included in the model as an additional spring constant of 50N/m. Similarly, from $t_a$ to $t_c$, the beam oscillates with natural frequency (200kHz), and the estimated spring constant is 25N/m. The model-based result including damping forces is shown in Figure 6.15. The higher $t_{AP}$ accuracy with experiment is found with squeezed-film air forces included in the calculation. The error is now reduced to 4.5% at 45V and 6.4% at 65V input.

As shown in Figure 6.14, the switch approaches the drain with a near-zero velocity at $t_c$. Applying the second pulse at this moment, the switch is maintained in the closed position. Figure 6.16 shows the experimental results of dual-pulse control at a 55V actuation voltage. The first pulse is kept similar to the one pulse exper-
6.5 Experiment evaluation

Figure 6.15: Comparison between experiment and calculation of duration $t_{ap}$ of the pulse to achieve near-zero velocity landing at $t_c$ with different actuation voltage $V_a$ on a 41V pull-in switch.

Due to imperfections in the fabrication process, the switch characteristics differ from device to device. Measured pull-in voltage varies from 35V to 48.5V, while the expected pull-in voltage is 41.6V for the described system. As previously discussed, the pull-in voltage and tip-gap relation is approximated by (6.22). Equivalent effective gaps were recalculated from the measured pull-in voltage. The dual-pulse parameters $V_a$, $t_{ap}$ or $t_c$ as shown in Figure 6.8 can be varied accordingly with measured pull-in. Practically, it is more feasible that the actuation voltage is kept
6.5 Experiment evaluation

Figure 6.16: Dual-pulse control to eliminate bouncing

constant. For this reason, new times $t_{ap}$ and $t_c$ of the dual-pulse shape were re-calculated with a fixed actuation voltage by (6.14), (6.19) and (6.21). Since the actuation voltage has to be at least equal to the highest pull-in voltage measured, 55V is chosen.

Again, a small change on the calculated pulse duration should be carried out for the calculated one-pulse actuation voltage to work experimentally. The value of calculated $t_{ap}$ was refined to achieve near-zero landing by observing the switch voltage output. The experiment was implemented on 16 switches with pull-in voltage varying from 35 to 48.5V. Figure 6.17 shows the comparison between calculation and experiment on the set of switches which have different pull-in voltages. The theory of adaptive control based on tip-gap versus pull-in voltage model is in good agreement with the experimental data.

For any switch the pull-in voltage is observed to drop over millions of cycles of step input actuation. The adaptive input shaping technique was tested on such a switch by calculating and measuring the voltage pulse width based on initial tip deflection estimates, then actuating the switch with a step input for millions of cycles to observe a significant drop in pull-in voltage. The pulse amplitude is once again computed and measured based on the new tip-gap estimate and this process is repeated over a range of decreasing pull-in voltages. The comparison between
6.5 Experiment evaluation

Figure 6.17: Adaptive control calculation versus experiment with near-zero landing velocity on 55V actuation voltage on a set of switches.

Figure 6.18: Adaptive control calculation versus experiment with near-zero landing velocity on 55V actuation voltage on a single switch with drifted pull-in voltage during testing.

calculation and experiment on the switch with drifted pull-in voltage is shown in Figure 6.18. The test was carried out for 500M cycles under cycling conditions with
6.5 Experiment evaluation

Figure 6.19: One dual-pulse input versus drifted pull-in voltage on the original 48V pull-in switch

frequency of 20kHz and 50% duty cycle. A similar trend of $t_{ap}$ and $t_c$ is found as was shown in Figure 6.17. This demonstrates that the proposed simple adaptive control approach can be used to recalculate an optimal pulse shape by periodic measurement of the pull-in voltage.

One question might arise: how often the three steps in adaptive control approach should be carried out? To address this question, two experiments were carried out on two switches. The dual-pulse control is designed for the initial measured pull-in voltage. Keeping the testing running with the fixed shaped input. Several switch voltage outputs were recorded in between as shown in Figure 6.19 and Figure 6.20. The first switch data in Figure 6.19 shows that at around 1V drop in pull-in voltage bouncing appears. Whereas at 2V drop in pull-in voltage, no bouncing is observed on the second switch (Figure 6.20). These output behaviour are not consistent. It is believed that the contact interaction, which is different from part to part, has an important effect. This issue is outside the scope of this work.
6.5 Experiment evaluation

Figure 6.20: One dual-pulse control versus drifted pull-in voltage on the original 38.5V pull-in switch

However, if it is compared with the behaviour of the switch under step actuation input as shown in Figure 6.4, the dual-pulse control on 2-3V pull-in drifted still improves the performance of the switch in terms of closing time and number of bounces. The frequency of recalculating the actuation voltage pulse width depends on the switching time specification, the rate of pull-in voltage change and the lifetime benefits which accrue from minimising bouncing. Further work is also required to determine a suitable method for measuring the drift in switch tip-gap as a function of switch cycles. The complexity of this measurement will also contribute to determining how frequently the actuation system should adapt the actuation pulse for optimum performance. The frequency of recalculating input shape depends on the output expectation and the rate of pull-in voltage change.
6.6 Summary

In this chapter, a closed-form analytical energy-based method was presented to accurately shape the input to eliminate bouncing while maintaining fast switching for a surface micro-machined DC contact MEMS switch. It was shown that the elimination of bouncing reduced switching time from 15$\mu$s to 2$\mu$s and should also improve switch reliability. An explicit solution for input shaping was presented for the switch. The computation is based on the conservation of energy principle with a Taylor expansion employed to generate an analytical solution. The analysis provides a ready to use calculation of the input shape of this open-loop control method. Simulink simulations and experimental results were found in excellent agreement with the calculation presented.

Furthermore, in order to better control parameter variation, an adaptive control method based on measured pull-in voltage to modify the timing and amplitudes of pulses has been developed in this work. A simple adaptive scheme has been developed and applied to the switch. This method uses a known relationship between pull-in voltage and tip-gap distances for switches with plastic deformation at the hinge anchor. It has been shown to be beneficial to the performance of both post-process and in-use switches with pull-in variation. The method can be expanded to other cases when the relationships of the system parameters to the pull-in voltage are known. It is expected that this explicit calculation and simple adaptive control methods will be deployed in real applications to improve the signal output and the life time of the switch.
Chapter 7

Conclusions and future work

*I would have written a shorter letter but I didn't have the time*

*Blaise Pascal, French Scientist (1623-1662)*

In this thesis, I have demonstrated that a complex MEMS ohmic switch can be modeled with an analytical model to predict instability phenomenon or a 2-D model for dynamic evaluations. An efficient adaptive control, which is based on lumped model, was also can be established analytically. In this chapter, the general concluding remarks and direction for future research will be presented.

7.1 Thesis summary

The research presented in this thesis has focused on the development of models, designs and control methodologies to reduce/remove the pull-in instability effect and high impact speed that have been reported as accelerating device failure mechanisms.

The thesis work was started by Chapter 2, which describes the state of the art of literature related to MEMS switches and their reliability issues. In Chapter 3, a novel generalized closed-form models for the pull-in instability position and pull-in voltage of a cantilever beam subjected to partial electrostatic actuation was developed. Chapter 4 inherits a model developed in Chapter 3 to propose an analytical
approach to design an electrostatically actuated cantilever structure without the pull-in instability issue. A dynamic behaviour of a MEMS ohmic contact switch including multi-asperities contact mechanics was presented in Chapter 5. Finally, the work of developing an open-loop and adaptive control based on lumped model to eliminate bouncing while maintaining fast switching for a surface micro-machined DC contact MEMS switch was demonstrated in Chapter 6. The principal outputs of the work include:

7.1.1 General pull-in instability model

The pull-in position was numerically calculated and interpolated to obtain an empirical formula for normalized pull-in position as a function of partial electrode length. The closed-form pull-in voltage expression was derived by employing Simpson's 3/8 rule for an implicit integral. The model takes into account a non-uniform distributed electrostatic force, partial electrode configuration and fringing field effects. The model is solely an analytical expression with no compensation factors required as have been used in previous works.

It was found that the pull-in stability position is independent with the beam structure, but varies with the electrode length. At full electrode length, the nominal pull-in position is around 0.44 while it reduced to 0.33 at short electrode positioned near the tip end of the beam. The analysis showed that with the variation of the pull-in voltage and pull-in position are very small as the electrode length varies from half to full length of the beam.

The developed model exhibits high accuracy for the pull-in voltage calculation in wide and narrow beams with partial electrode overlap. The accuracy of the model was validated through comparison with Finite Element Method (FEM) simulation results and correlation to within 6.6% error was obtained. The model was also validated against other existing empirical and analytical models and demonstrated to be of equal accuracy with a more generalized model framework.
7.1 Thesis summary

7.1.2 Design to avoid pull-in instability

Following this guideline, a switch can be designed such that it will close contact before the instability occurs. This in turn will translate into a reduced contact speed and force. The results based on the proposed analytical method were validated and found to agree well with FEM simulation results with maximum 5% error. The proposed guidelines can be instrumental in general MEMS devices design where the stable range of travel is an important factor. It was shown that if the electrode is positioned near the anchor with the length equals to half of beam length, the cantilever structure can travel the full gap without instability effect. However, the full range travel is paid off by the pull-in voltage. The optimal design can be found by plotting a combined graph which shows pull-in voltage and tip deflection at pull-in which provides a relatively fast and straightforward procedure for a MEMS designer's prospective.

7.1.3 Integrated dynamic model with contact mechanics and degradation analysis

The rough surfaces of switch contacts were represented by a number of asperities. The statistical distribution of the asperities was assumed to be Gaussian distribution. As the asperities have different heights, the beam only interacts and creates deformation on several of the highest asperities. The contact model of 24 asperities with identical radius $R=0.05\mu m$ and Gaussian heights was used to represent the Ru-Ru contact in the switch.

The multi-asperities model was then incorporated into the dynamic model of an ohmic-contact electrostatic MEMS switches. The model also takes into account other fundamental components of real switch geometry, squeeze-film damping effect and electrostatic. The whole system model was simulated by using the finite difference method (FDM).

The model was used to simulate the deformation of the contact, the switching speed, the tip displacement and the bouncing behavior of the switch. An evaluation
of the dependence of contact resistance on contact roughness was presented. The predicted contact resistances agree well with the measured switch resistances versus actuation voltages. The simulation results for closing time, the number and duration of bounces, the contact deformation and the settling time are in excellent agreement with the experimental results for a gold beam, ruthenium contact MEMS ohmic switch.

The FDM approach was then used to analyse the effect of different device geometries on lifetime and reliability which could not be done on the basis of an empirical fit or analyses based on existing analytical models. The proposed method is also less time consuming than a full 3-D model possible with Finite Element Method or Molecular Dynamics. The work demonstrated that the mechanical (plastic) deformation of the switch is the dominant factor in the actuation voltage degradation.

7.1.4 Energy-based approach to adaptive pulse shaping control

A closed-form analytical formulation that accurately shapes the switch actuation voltage waveform to eliminate bouncing while maintaining fast switching for a surface micro-machined DC contact MEMS switch was developed. The computation is based on the conservation of energy with a Taylor series approximation. The analysis provided a ready to use calculation of the input shape of this open-loop control method. Simulink simulations and experimental results are in excellent agreement with the calculation presented. The highest error was found at 6.5% at actuation voltage 45V input. The elimination of bouncing can reduce switching time from 15\mu s to 2\mu s and should also improve switch reliability.

To better control the switch dynamics in the presence of switch parameter variation, an adaptive control which is based on measured pull-in voltage to modify the timing and amplitudes of pulses of the open-loop control was developed. The method uses a known relationship between pull-in voltage and tip-gap distances for switches with plastic deformation at the hinge anchor. Validation was implemented on set of 16 switches with pull-in voltage varying from 35 to 48.5V and found to be
7.2 Contributions of the work

in good agreement with experimental results. It was shown to be beneficial to the performance of both post-process and in use switches with pull-in variation.

7.2 Contributions of the work

The outcomes of the proposed methodologies have contributed new scientific findings and help to extend the knowledge of the fields as described in the following:

7.2.1 Contributions on closed-form pull-in model

The work advances the closed-form pull-in model with a more generalized framework. To the author's knowledge, this is only the second published work that deals with cantilever structures under electrostatic force with geometric electrode overlap variation. Unlike the previous work, this work employs a non-linear formulation for the stiffness of the beam with higher accuracy than previously reported and confirmed with FEM simulations. Following this analysis, the pull-in stability position is found to be independent of the beam structure, but varies with the electrode length. The developed method also has the advantage of quantifying the beam deflection at pull-in without the need for FEM simulation.

7.2.2 Contributions on electrostatic design structure without instability

The leveraged bending method which extends the travel range of electrostatic MEMS devices was initially developed in 1999. Since then, the method has been widely employed. This work is the first closed-form analytical analysis to guide the design of cantilever structures with extended travel range and no pull-in instability. In this thesis the method has been demonstrated for design of a MEMS cantilever based switch.
7.2 Contributions of the work

7.2.3 Contribution on dynamic modeling of MEMS ohmic switch

In this work, the multi-asperities nonlinear contact with adhesion is successfully integrated into a dynamic 2-D model of the switch for the first time. By including the contact mechanics, the switch dynamic performance (e.g. contact resistance, switch bouncing effects) can be captured with higher accuracy. This simulation method can be used as a good tool for evaluating device failure mechanisms and to develop control methods to improve the operation (such as remove bouncing effects).

The developed model can be a useful tool in understanding the complex phenomenon of contact interaction and issues concerned with contact material. It can also be used as a design tool to evaluate the dynamic behavior of a certain MEMS ohmic switch design. The switch degradation effects during life-time operation can also be evaluated based on the developed dynamic model.

7.2.4 Contribution on open-loop control method to reduce contact speed and remove bouncing

The open-loop control method based on energy conservation and dual-pulse waveform has been investigated since 2006. However, all previously published works were based on numerical or semi-analytical analysis to find the parameters of the dual-pulse waveform. Furthermore, many practical factors were not taken into consideration. This work presents a novel analytical method to develop the waveform for open-loop control. The method includes all of the most important practical effects such as: damping, practical actuation pulse in the calculation. The developed method has been validated showing close agreement with experiment measurements.

Furthermore, the method can be applied in an adaptive control strategy which captures the degradation effects of the switch during operation. The analytical formulation can be easily modified to adapt with the variation of system parameters. The general method which is based on the energy conservation rule can be developed for many other electrostatically actuation structures.
7.3 Future work

Some possible directions for future research are outlined below:

7.3.1 Models for other electrostatically actuated structure

All the developed methodologies were specially developed for cantilever MEMS ohmic structures. The nonlinear stiffness analysis can be applied to fixed-fixed beam structures under electrostatic force to achieve a closed-form pull-in model. Hence, a similar analysis of leveraged bending with analytical formulation, as presented in this work, can be developed to extend the travel range of fixed-fixed structures.

7.3.2 Contact model

An improved model of the contact dynamics that takes into account the rough surface with permanent deformation versus cycles can be developed in the future work based on this work. A higher accuracy model of adhesion forces should be developed to capture the loading and unloading difference for the full cycle of contacting. As previously mentioned, the hot switching condition is an important factor that affects the reliability of the contact. A model that can simulate the hot switching condition based on multi-asperities is also a possible future development for the contact model.

7.3.3 Reliability evaluation

The developed open-loop control method has been validated on a MEMS ohmic switch reducing contact speed so as to remove the bouncing issue. This is believed to improve the reliability of the switch. However, the lifetime testing based on the dual-pulse open-loop control method have not been implemented yet. Future work will evaluate the developed method to compare and contrast the effectiveness of the method compared with normal actuation (square waveform).
Appendix A

MATLAB script

In this appendix, the Matlab simulation scripts for the electrostatically actuated cantilever MEMS ohmic switch as discussed in Chapter 5. The formulations are based on using 2D Finite Difference Method to solve an equation of motion based on Euler-Bernoulli theory.

A.1 Formulations

Figure A.1(a) shows the top view of the SEM micrograph of a SPST ruthenium contact RF-MEMS switch used in this work.

As the beam is non-uniform the schematic of FDA as shown in Figure 5.2 is modified as shown in Figure A.2. The beam is divided by N nodes. The gate is positioned beneath the wide cross-section, and the nodes in this area are marked from \( i_{\text{start}} \) to \( i_{\text{end}} \). These nodes have higher value of second moment of inertia (\( I_2 \)). Nodes from \( i_{\text{end}}+1 \) to \( N \) have narrow cross-section but are assumed to be \( I_2 \) for simplification, as they have very low effect on the stiffness of the beam. Nodes 1 to \( i_{\text{start}}-1 \) are \( I_1 \). Only nodes from \( i_{\text{start}} \) to \( i_{\text{end}} \) have electrostatic force (\( f_e \)), while all nodes have damping force (\( P_d \)). When the deflection of the beam tip (node \( N \)) reaches \( t_g \), the contact interaction will be taken into account.
Figure A.1: (a) SEM micrograph of the non-uniform RF-MEMS switch. (b) Schematic side view of the switch.

Figure A.2: Schematic of non-uniform cantilever beam switch using Finite Difference Method.
A.2 MATLAB script

The dynamic beam equation consists of electrostatic force equation and squeeze-film damping formulation is recited from (5.1) as

\[ -\varepsilon I \frac{\partial^4 y}{\partial x^4} + f_c - P_d = m \frac{\partial^2 y}{\partial t^2} \]  

(A.1)

The electrostatic force, damping and boundary conditions have been described thoroughly in Chapter 5, therefore it is not necessary to repeat here. Numerical solution for differential equation (A.1) by finite difference method \cite{159,160} for M-file programming in MATLAB is shown in next section.

A.2 MATLAB script

```matlab
% (non_uniform_switch_with_24_asperites.m) MATLAB script to solve
% differential equation of dynamic electrostatically actuated MEMS
% ohmic switch.
% Model includes
% Non-uniform cantilever beam
% Electrostatic force
% Squeeze-film damping
% Multi-asperities contact mechanics

% Cuong Do - Cork Institute of Technology
% 21/4/2012

%% Set default parameters
% Clean everything before running new program
clear
clc

set(0,'defaultAxesFontName', 'Times New Roman')
set(0,'defaultTextFontName', 'Times New Roman')
```

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set(0,'defaultTextFontSize', 16)
set(0,'defaultAxesFontSize', 16)

% Set the total time simulation (micro-seconds)
T = 30e-6;

% Set number of subinterval along the x axis (along the beam length)
N = 22;

% Set the time step resolution, dt = 1ns
dt = 1e-9;

% density of beam material
p = 19300;  %Gold

% effective Young's modulus of beam material
E = 78e+9;  % Gold

% beam width
w_2 = 38e-6;  % wide cross-section
w_1 = 14e-6;  % narrow cross-section near anchor

% beam thickness
h_t = 5.4e-6;

% real beam length
L = 88e-6;

% plus 2 imaginary nodes used for simulation purpose.
% Do not change this.
L = L + 2*L/N;

% initial spacing between tip and drain
tg = 0.26e-6;
% initial beam-gate spacing 
d = 0.6e-6;

% permittivity of free space 
e_0 = 8.8542e-12;

dx = L/N; % length of 1 node
x = [0:N+1]*dx; % position along x axis of nodes
M = T/dt; % total number of iteration times
t = [0:M]*dt; % time point

% Second moment of the cross-section w*h^3/12
I_2 = (w_2*h_t*h_t*h_t)/12;
I_1 = (w_1*h_t*h_t*h_t)/12;

% Set parameter of multi-asperity contact

% Pi number 
Pi = 3.14;

% Poisson's ratio of contact material 
p_r = 0.30; % Ruthenium

% Surface energy of contact material 
s_e = 4.2; % Ruthenium

% Curvature radius of asperity 
R = 0.5e-7; % all asperities are assumed to be identical

% Density: number of asperity per um^-2 
D_s = 20;

% standard deviation of asperity height: rms roughness
A.2 MATLAB script

\begin{verbatim}

sigma=4e-9;  \% 4 nm

\% nominal contact area \( \text{um}^2 \)
A_n=1.2;

\% Young modulus of contact material
E_g = 414e+9;

\% air viscosity
a_v = 1.86e-5;

\% effective Young modulus (Herzt modulus) of contact material
E_ge=1/(2*(1-p_r^-2)/E_g);  \% Ruthenium-Ruthenium

l_e=5e-9;  \% electron mean free path of Ruthenium
resi=71e-9;  \% resistivity of Ruthenium

\% All asperities \( \text{D*A_n} = 24 \) asperities with heights follow 
\% Gaussian distribution
z(1:24)=[0.5*sigma,0.5*sigma,0.5*sigma,0.5*sigma,0.5*sigma,0.5*sigma,
          0.5*sigma,0.5*sigma,0.5*sigma,0.5*sigma,0.5*sigma,0.5*sigma,
          0.5*sigma,0.5*sigma,0.5*sigma,0.5*sigma,0.5*sigma,0.5*sigma,
          1.5*sigma,1.5*sigma,1.5*sigma,1.5*sigma,1.5*sigma,1.5*sigma,
          2*sigma,2*sigma,2.5*sigma,2.5*sigma,2.5*sigma,2.5*sigma,3*sigma];

N_a=24;  \% number of asperities.

\% Voltage apply
\% \% Step voltage
V(1:M+1) = 50;

\% \% Ramp voltage
\%V(1:M+1)= linspace(50,80,M+1);

\end{verbatim}

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A.2 MATLAB script

% % simplified dual-pulse control 60 - 50V
% % start
% V(1:200) = linspace(0,60,200);
% V(201:795) = 60;
% V(796:995)= linspace(60,0,200);
% V(996:1550) = 0;
% V(1551:1750) = linspace(0,50,200);
% V(1751:M+1) = 50;
% % % end

% Reserve variables spaces before running numerical to save running time
u = zeros(N+2,M+1);  % node deflection
h = zeros(N+2,M+1);  % substrate-to-node gap
P = zeros(N+2,M+1);  % node damping
P_s = zeros(N+2,M+1);  % node damping converted
fe = zeros(N+2,M+1);  % node electrostatic force

% set overlapped nodes over electrode. If number of node (N) changed, these values should be changed accordingly.
% i_start
i1 = 5;

% i_end
i2 = 19;

% mass per unit length
m = p*w_2*h_t;  %wide section
m_s = p*w_1*h_t;  %narrow section

r1 = -(E*I_2*dt*dt)/(dx*dx*dx*dx*m);
r1_s = -(E*I_1*dt*dt)/(dx*dx*dx*dx*m_s);
A.2 MATLAB script

\[
\begin{align*}
r2 &= dt*dt/m; \\
r2_s &= dt*dt/m_s; \\
% Set values for initial step (time=0). It is assigned to be 1 \\
% because there is no "zero" index in MATLAB \\
for i = 1:N+2 \\
    u(i,1) &= 0; \quad % i position at t=1 \\
    h(i,1) &= d - u(i,1); \\
    P(i,1) &= 0; \\
    P_s(i,1) &= ((2*w_2)/3)*P(i,1); \\
end \\
% Set values for first node x=1 (anchor is fixed) \\
for k = 1:M+1 \\
    u(1,k) &= 0; \quad % 1st position for all k \\
    h(1,k) &= d - u(1,k); \\
    P(1,k) &= 0; \\
    P_s(1,k) &= ((2*w_2)/3)*P(1,k); \\
end \\
% Due to the boundary condition at t=1, the second time step \\
% should be calculated separately \\
% electrostatic forces at t=1 \\
% fe depends on the node number \\
for i = 1:N \\
    if (i<i1) | (i>i2) \\
        fe(i,1) &= 0; \\
    end \\
    if (i==i1) | (i==i2) \\
        fe(i,1) &= (e0*V(1)*V(1)*w_2)/(4*(d-u(i,1)).^2); \\
    end \\
    if (i>i1) & (i<i2) \\
        fe(i,1) &= (e0*V(1)*V(1)*w_2)/(2*(d-u(i,1)).^2); \\
end
\end{align*}
\]
end
end

% Deflection and damping at t=2
u(2,2) = (r1_s/2)*(-4*u(1,1)+7*u(2,1)-4*u(3,1)+u(4,1))
      + (fe(2,1)+P_s(2,1))*r2_s/2 + u(2,1);

h(2,2)= d - u(2,2);

u(3:i1,2) = (r1_s/2)*(u(1:i1-2,1)-4*u(2:i1-1,1)+6*u(3:i1,1)
      -4*u(4:i1+1,1)+u(5:i1+2,1)) + (fe(3:i1,1)+P_s(3:i1,1))*r2_s/2
      + u(3:i1,1);

u(i1+1:N,2) = (r1/2)*(u(i1-1:N-2,1)-4*u(i1:N-1,1)+6*u(i1+1:N,1)
      -4*u(i1+2:N+1,1)+u(i1+3:N+2,1)) + (fe(i1+1:N,1)+P_s(i1+1:N,1))
           *r2/2 + u(i1+1:N,1);

h(3:N,2)= d - u(3:N,2);
P(2,2) = 0;

P(3:N,2)= (18*a_v*dx^2)/(dt*h(3:N,2).^3)*(h(2:N-1,2)-h(2:N-1,1))
       - (3/(2*h(3:N,2)))*(h(3:N,2)-h(1:N-2,2))*(P(2:N-1,2)
       -P(1:N-2,2)) + ((12*dx^2)/w_2^2 + 2)*P(2:N-1,2) - P(1:N-2,2);

P(N,2) = 0;
P_s(2:N,2)=((2*w_2)/3).*P(2:N,2);

% Electrostatic forces at t=2
% fe depends on the node number
for i = 1:N
    if (i<i1) | (i>i2)
        fe(i,2)=0;
    end
    if (i==i1) | (i==i2)
A.2 MATLAB script

```matlab
fe(i,2) = (e0*V(2)*V(2)*w_2)/(4*(d-u(i,2)).^2);
end
if (i>i1) & (i<i2)
  fe(i,2) = (e0*V(2)*V(2)*w_2)/(2*(d-u(i,2)).^2);
end
end
vel_tip(l)=0;
vel_tip(2)=(u(N,2)-u(N,1))/dt;

% Iteration calculation of electrostatic force, beam deflection, % damping at t=3 to M+1
for k = 3:M+1
  u(2,k) = (rl_s/2)*(-4*u(1,k-1)+7*u(2,k-1)-4*u(3,k-1)+u(4,k-1))
  + (fe(2,k-1)+P_s(2,k-1))*r2_s/2 + u(2,k-1);
  u(3:i1,k) = rl_s*(u(1:i1-2,k-1)-4*u(2:i1-1,k-1)+6*u(3:i1,k-1)
  -4*u(4:i1+1,k-1)+u(5:i1+2,k-1)) + (fe(3:i1,k-1)
  +P_s(3:i1,k-1))*r2_s + 2*u(3:i1,k-1)-u(3:i1,k-2);
  u(i1+1:N,k) = rl*(u(i1-1:N-2,k-1)-4*u(i1-1:N-1,k-1)+6*u(i1+1:N,k-1)
  -4*u(i1+2:N+1,k-1)+u(i1+3:N+2,k-1)) + (fe(i1+1:N,k-1)
  + P_s(i1+1:N,k-1))*r2 + 2*u(i1+1:N,k-1)-u(i1+1:N,k-2);
  h(2:N,k)= d - u(2:N,k);
  P(2,k) = 0;
  P(3:N,k)= (18*a_v*dx^2)/(dt*h(3:N,k).^3)*(h(2:N-1,k)
  -h(2:N-1,k-1)) - (3/(2*h(3:N,k)))*(h(3:N,k)-h(1:N-2,k))
  *(P(2:N-1,k)-P(1:N-2,k)) + ((12*dx^2)/w_2^2 + 2)
  *P(2:N-1,k) - P(1:N-2,k);
  P(N,k) = 0;
  P_s(2:N,k)=3*((2*w_2)/3)*P(2:N,k);
for i = 1:N
```

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if (i<i1) | (i>i2)
fe(i,k)=0;
end
if (i==i1) | (i==i2)
fe(i,k) = (e0*V(k)*V(k)*w_2)/(4*(d-u(i,k)).^2);
end
if (i>i1) & (i<i2)
fe(i,k) = (e0*V(k)*V(k)*w_2)/(2*(d-u(i,k)).^2);
end

% calculate imaginary position using boundary
% condition N(L) bending moment
u(N+1,k) = 2*u(N,k)-u(N-1,k);

% calculate imaginary position using boundary
% condition V(L) shear force
u(N+2,k) = 4*u(N,k)-4*u(N-1,k)+u(N-2,k);

% calculation the velocity of tip
vel_tip(k)=(u(N,k)-u(N,k-1))/dt;

% check bouncing condition
if (u(N,k) <= tg)
  % not reach to contact yet
  u(N+2,k) = 4*u(N,k)-4*u(N-1,k)+u(N-2,k);
else
  % Touching to the contact. Begin to calculate the interaction
  % force for separate asperity
  w_a = u(N,k)-tg;
P_a(1:24)=zeros(1,24);
fs=0;
rad_t=0;
Res(k)=0;  % initial resistance value

for i=1:24  % check the interference of into each asperity
    rho(i)=z(i)-(3*sigma-w_a);

    if rho(i)>=0  % if there is interference with asperity i
        rad(i)=sqrt(rho(i)*R);  % radius

        % Resistance calculation
        Res_a(i)=4*resi*l_e/(3*Pi*rad(i)^2)+l_e/rad(i)
        *resi/(2*rad(i));

        Res(k)=Res(k)+(l/Res_a(i));

    end
end

% total contact resistance
Res(k)=1/Res(k);

% total force
force(k)=fs;

% contact boundary condition at the tip end (node N)
r3 = (fs*2*dx^3)/(E*I_2);
    u(N+2,k) = 4*u(N,k)-4*u(N-1,k)+u(N-2,k)+r3;
end
end

% plot the tip deflection
figure
plot(t*10e5,-u(N,:)*10e5)
ylabel('Tip Displacement [micro-metre]')
xlabel('Time [micro-second]')

% if step actuation was applied
str=sprintf('Step actuation voltage = %d', V(1));
title(str)

% plot the contact resistance
figure
plot(t*10e5,Res)
ylabel('Contact resistance [Ohm]')
xlabel('Time [micro-second]')
Appendix B

Spring constant and pull-in voltage models of non-uniform electrostatically actuated cantilever structure

In Chapter 3, the high accuracy generalized closed-form pull-in voltage was introduced. The model is applicable for uniform cantilever beam structure. However, in practice, in order to reduce the pull-in voltage, the MEMS cantilever beam is usually designed with non-uniform structure, which is narrower for the area close to the anchor and wider for the electrode area. The non-uniform design is found in several works [66][88], the switch which was used in Chapter 5 and 6 of this work also has this type of design. Therefore, the model developed in Chapter 3 is not applicable for some practical designs which have non-uniform structure.

The method which was used in Chapter 3 can be applied to form a closed-form model for non-uniform structure. But an analytical will become very complex. Hence, in this appendix, a simple comprehensive spring constant and pull-in voltage models for non-uniform electrostatically actuated cantilever structure is presented. The method is based on a basic uniform load over small deflection mechanism of the cantilever structure.
B.1 Formulations

B.1.1 Spring constant

Figure B.1(a) shows a schematic of a non-uniform cantilever-based MEMS switch. The width of the near-anchor area is narrower than the width over the electrode area. The superposition method [100] is used to formulate the spring constant of the structure. Figure B.1(b) show the discretization of the beam into 3 sections with four points (A, B, C, D). The second moment of inertia (I) of 3 sections are shown on the figure. The electrostatic force is only applied on BC section and assumed to be uniform load (q).

Based on the superposition method, each section is considered separately as showed in Figure B.2. Point B in Section BC (Figure B.2(a)) is assumed to be fixed for initial calculation. Uniform load (q) pushes the section BC downward. The
deflection and angle at point C at initial calculation are [100]

\[
\delta_C = \frac{q b^4}{8 E I_2} \quad \text{(B.1)}
\]

\[
\theta_C = \frac{q b^3}{6 E I_2} \quad \text{(B.2)}
\]

where \( E \) is Young’s modulus of the beam material.

Point B in section AB (Figure B.2(b)) is assumed to be free end. On this section, the shear force \((V = q b)\) and the moment \((M = q b^2 / 2)\) pushes it downward. Deflection and angle of point B are [100]

\[
\delta_B = \frac{q b a^2 (4a + 3b)}{12 E I_1} \quad \text{(B.3)}
\]

\[
\theta_B = \frac{q b a (a + b)}{2 E I_1} \quad \text{(B.4)}
\]

Superimpose section BC over section AB (Figure B.3), assuming the angle \( \theta_B \) is
Figure B.3: Sections AB, BC superimposition

very small, the deflection and angle of point C are now becomes

\[(\delta_C)_2 = \delta_B + \delta_C + b\theta_B = \frac{qb^3}{8EI_2} + \frac{qba^2(4a + 3b)}{12EI_1} + \frac{qba(a + b)}{2EI_1}\]  \hspace{1cm} (B.5)

\[(\theta_C)_2 = \theta_B + \theta_C = \frac{qb^3}{6EI_2} + \frac{qba(a + b)}{2EI_1}\]  \hspace{1cm} (B.6)

The section CD has no force applied, therefore, it only acts as a lever. Hence, the deflection at the tip of the beam (point D) under uniform load can be found as

\[\delta_D = (\delta_C)_2 + c(\theta_C)_2 = \frac{qb^3(3b + 4c)}{24EI_2} + \frac{qba(4a + 3b) + 6(a + b)(b + c)}{12EI_1}\]  \hspace{1cm} (B.7)

The effective spring constant (linear) is defined as total force over total deflection, one obtains

\[k = \frac{\text{Force}}{\text{Deflection}} = \frac{qb}{\delta_D} = \frac{24EI_1I_2}{I_1[b^2(3b + 4c)] + 2I_2a[4a + 3b] + 6(a + b)(b + c)}\]  \hspace{1cm} (B.8)

It is shown that the value of \(I_3\) has no contribution on the spring constant of the beam
B.1 Formulations

B.1.2 Pull-in voltage

As described in Chapter 2, the pull-in position of the lumped model is always at one-third of the gap ($1/3g$). However, the analysis presented in Chapter 3 and 4 pointed out that the instability point in real cantilever structure is higher than $1/3g$. Follow the descriptions in Chapter 4, if the ratio $b/a$ is not very small ($b/a > 0.5$), the normalized pull-in instability position at the tip of beam shown in Figure B.1 can be simplified as

$$y_{pi} = 0.44 \left(1 + 1.22 \frac{c}{a + b}\right) \quad (B.9)$$

Therefore, the tip deflection at the instability point is, $(\delta_D)_{pi} = gy_{pi}$. The electrostatic force at that beam deflection can be approximated as (load concentrates in the middle of the section BC)

$$F_e = \frac{\varepsilon_0 w_2 b V^2}{2(g - \beta(\delta_D)_{pi})^2} \quad (B.10)$$

with $\beta = (a + b/2)/(a + b + c)$ and $w_2$ is width of the overlapped electrode area (section BC)

Equating the electrostatic force in (B.10) with the restoring force ($F_s = k(\delta_D)_{pi}$) at the instability point, the pull-in voltage for non-uniform structure shown in Figure B.1 can be found as

$$V_{pi} = \sqrt{\frac{2kg^3 y_{pi}(1 - \beta y_{pi})^2}{\varepsilon_0 w_2 b}}, \quad (B.11)$$

If the beam tip to drain gap ($d$) is smaller than the pull-in point ($d < gy_{pi}$), the beam closes before the instability occurs. The following calculation should be used

$$V_{pi} = \sqrt{\frac{2kd(g - \beta d)^2}{\varepsilon_0 w_2 b}}, \text{ for } d < gy_{pi} \quad (B.12)$$

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Table B.1: $E=57\text{GPa}$, $a=20\mu\text{m}$, $b=180\mu\text{m}$, $c=0$, $w_1 = w_2 = 50\mu\text{m}$, $t=3\mu\text{m}$, $g=2\mu\text{m}$

<table>
<thead>
<tr>
<th>Models</th>
<th>$V_{pn}$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM simulation</td>
<td>15.91</td>
</tr>
<tr>
<td>Chapter 3 (3.14)</td>
<td>16.29</td>
</tr>
<tr>
<td>Ref. [94]</td>
<td>16.57</td>
</tr>
<tr>
<td>This work (B.11)</td>
<td>17.13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Compared</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error between (3.14) and FEM</td>
<td>2.3</td>
</tr>
<tr>
<td>Error between [94] and FEM</td>
<td>3.9</td>
</tr>
<tr>
<td>Error between (B.11) and FEM</td>
<td>7.1</td>
</tr>
</tbody>
</table>

Note that the fringing field effects as described in Chapter 3 should be included in (B.11) and (B.12) for higher accuracy, especially in case of small $w_2/g$ ratio. The validation in the next section neglects this effect. The beam stiffness model does not account the non-uniform load of electrostatic force. Furthermore, the beam shape function of non-uniform structure might slightly different from uniform switch, therefore the accuracy of the pull-in instability point in (B.9) is not very accurate especially when the width of the small section is very narrow compared big section ($w_1 \ll w_2$). Hence the accuracies of (B.11) and (B.12) are not expected to be high due to those limitations.

### B.2 Model validations

Table B.1 shows the pull-in voltage comparisons of (B.11) and other models versus FEM simulation result for uniform beam ($w_1 = w_2 = 50\mu\text{m}$). The results of uniform beam as shown in Figure 3.11 is extracted at $\alpha = 0.9$.

It is shown that the model developed in Chapter 3 predicts the closest value of only 2.3% error with FEM result. The model developed in (B.11) is 7.1% error with the FEM result. The accuracy is found to be less than the models presented in Chapter 3 and in [94]. However, the model in (B.11) can be applicable for a wider range of structures compared with the other two models.

Table B.2 shows the comparison between this work and the model developed by
B.2 Model validations

Sara et al. [161] for non-uniform beam with c=0. The FEM results were simulated from CoventorWare by Sara et al. in [161]. It shows that the model in this work is in excellent agreement compared with FEM simulation result. As discussed in the previous section, the model in (B.11) exists several limitations, therefore the near-zero errors as shown in Table B.2 might be due to the rough meshing in FEM simulation or just a coincidence.

Table B.3 shows the comparison between (B.11), FEM (ANSYS) and measured data on the non-uniform switch used in Chapter 5 and 6 (Figure 5.5). The nominal designed details were described in Section 5.3. FEM simulations from ANSYS for this switch were carried out in Chapter 6. Follow (B.9) the instability point of the beam structure is \((\delta_D)_p = 0.53\mu m\). It is higher than the designed tip gap \((d=0.26\mu m)\). Hence, the beam reaches to the contact pad before the instability occurs. Therefore, (B.12) is used to find the pull-in voltage. There are good agreements between (B.12) and FEM results for switch used in this dissertation with errors less than 2% in both spring constant and pull-in voltage.

Table B.3: Nominal designed parameters of the switch in Chapter 5, \(E=78\) GPa, \(a=14\mu m, b=63\mu m, c=14\mu m, w_1 = 14\mu m, w_2 = 38\mu m, t=6\mu m, g=0.6\mu m, d=0.26\mu m\)

<table>
<thead>
<tr>
<th>(w_2) ((\mu m))</th>
<th>(V_p (V)) (FEM)</th>
<th>(V_p (V)) (B.11)</th>
<th>FEM vs. [161] (%)</th>
<th>FEM vs. (B.11) (%)</th>
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</thead>
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<td>100</td>
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<td>11.3</td>
<td>9.5</td>
<td>11.42</td>
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Bibliography


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