Teaching of Structural Analysis into the Future

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ABSTRACT: The curricula of modern engineering programmes achieve a greater number of learning outcomes and cover a broader range of subject areas than ever before. This has resulted in a reduction in the hours that are available to teach structural engineering. At the same time the work of graduate structural engineers has changed and is likely to change further in the future. This paper considers what the kernel of essential knowledge for structural engineering should contain. More specifically it explores what elements of this kernel must be taught in university. The paper does not result in a definitive list of topics but makes some initial suggestions and promotes a rationale by which such a list might be arrived at.

The paper argues that it is import to acknowledge that much of structural engineering analysis is pragmatic. Many of the basic theories are simplifications that are useful only in certain circumstances with certain materials. This complicates identifying a small set of structural engineering rules, Newton's Laws excepted. While structural engineers should have knowledge of mechanics of solids, elasticity and methods of analysing statically indeterminate structures the level of complexity that needs to be achieved is not immediately clear.

Modern structural engineering practice suggests that some areas of structural engineering analysis, such as the flexibility method, are obsolete; however, some of these methods are useful for exploring important concepts and developing qualitative analysis skills. Qualitative analysis skills are vitally important because most structural analysis is performed using computer software and it is essential that an engineer is able to critique the output from such programs. Qualitative analysis is a key skill in structural design.

The paper also considers how the role of structural engineers is likely to change with the increased use of artificial intelligence and machine learning, and with the development of parametric modeling packages that allow engineers to vary the form of a structure and observe the changes in structural response instantly.

The paper also considers whether there is a need for different objectives when it comes to selecting topics that should be taught at undergraduate and at postgraduate level. Undergraduate curricula should ensure basic competence: equilibrium of forces, the relationship between stresses and strains and knowledge of the failure mechanisms for different materials and structure type etc. Should postgraduate curricula be designed to ensure that knowledge of a wide variety of specialist techniques, such as: fracture mechanics, classical elasticity, continuum mechanics, structural optimisation, design and analysis of plates and shells etc. are maintained within the engineering profession? From this basis the paper tries to address the core competences that every structural engineer should have as well as the core knowledge that would be essential for students to further develop their knowledge of structural engineering once they have graduated.

KEY WORDS: Engineering education; Structural Engineering; Structural analysis; Structural design; Engineering Curriculum; Future.

1 INTRODUCTION

This paper is intended to be the starting point in a discussion on what should be included in current and future structural engineering curricula.

The engineering curriculum has always been full. In the past civil and structural engineers studied a wide variety of technical subjects within the broad field of engineering science and spent many hours working on design projects. Modern engineering curricula try to cover these areas while also dedicating more time to structured group-work, report writing and communication skills. Although students spend longer in college, the number and range of technical fields that students must master is considerably broader than in the past. As a result the time available to teach structural engineering has reduced. At the same time the work of a structural engineer has changed and some methods of analysis and design that were traditionally taught in college are rarely used in practice. As a result the structural engineering curriculum is changing. This paper considers what the core curriculum for structural engineering should contain.

This is not a trivial question. While some traditional analysis methods, such as graphic statics, can be omitted without significant consequences the loss of other types of hand analysis, such as influence line diagrams, may hinder students developing a full understanding of structural behavior. Similarly, while all structural and civil engineering students must have a knowledge of mechanics of solids, what aspects of mechanics of solids are core? As structural engineers embrace an ever wider variety of materials should mechanics of solids and mechanics of materials be studied in greater detail? Should the emphasis on linear-elastic matrix methods move from teaching the basic algorithms to an increased emphasis on understanding and overcoming the limitations of such methods? Where does plastic analysis fit when most structural analysis is performed using linear-elastic theory? What will the core skills of a civil/structural engineer be when structural designs can be developed by autonomous algorithms that take 3D general arrangement models as their input?

2 STRUCTURAL ANALYSIS & PRAGMATISM

It is possible to interpret the history of the development of structural engineering as the gradual development of ever more sophisticated mathematical models [1,2,3,4,5,6]. This interpretation is correct but it is not necessarily complete. This is an important point to consider because it is easy to imagine that the core canon of structural engineering is fixed. This is not necessarily the case.

2.1 Engineers Bending Theory

Consider engineers' bending theory, which is arguably among the most useful structural engineering theories. It began with the work of Galileo, and was developed by Marriott, James and Daniel Bernoulli, Euler, Coulomb, Hodgkinson, Navier and others. Figure 1 shows a sequence of assumptions of how the internal longitudinal stresses are distributed in a cross-section of the base of a cantilever [3].

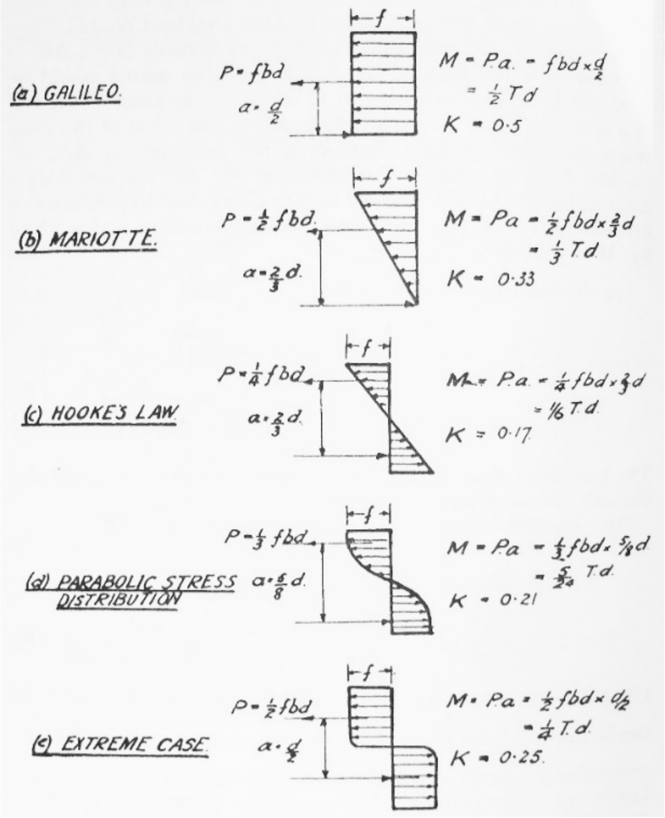

Figure 1. Stress distributions and calculated moment capacity of a cantilever [3].

The wonder to a modern engineer is that the final "correct" solution took so long to emerge. However, engineers today have the benefit of hindsight and typically use structural materials that have well defined properties. The profession is so comfortable with engineers' bending theory that it is easy to forget that this theory is a combination of three separate concepts. These are:

1. The internal forces in a beam must be in equilibrium with the externally applied loads.
2. The internal forces developed in the beam are related to internal deformations of the material in the beam.
3. The beam fails when the internal forces (stresses) or internal deformations (strains) exceed the capacity of the material the beam is formed from.

The first point, which was understood by Galileo and was given in a general form by Coulomb in the 1770s is clear today but was poorly understood for many years [7]. Coulomb specified the need for the internal forces along the section A-D in Figure 2 to be in equilibrium with the applied load. He stated that the longitudinal stresses must develop an internal moment and that there was a need for forces with a vertical component to counteract the vertical load. However, the discussions following James Barton's paper on the Boyne Viaduct to the ICE in 1855, shortly before Jourawski's work on shear was published in 1856, shows how the shear forces in a beam were poorly understood [8,9].

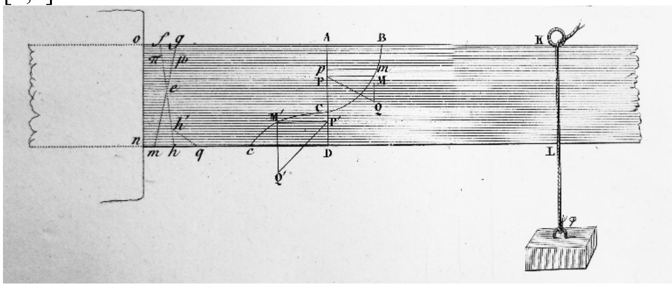

Figure 2. Equilibrium of the internal forces and external loads [7]

Regarding the second point, the relationship between stress and strain in a beam made from an elastic material is far more complex than $\sigma = E\epsilon$. This formula and the familiar $\sigma = \frac{My}{I}$ work well for simple steel universal beams but with larger plate girders effects such as shear lag must be considered. The third concept, that of material failure criteria is also treated in a very simple manner when designing steel beams. It is notable that when designing other components it is often essential to consider the full stress and strain tensors and to take account of fracture mechanics and fatigue.

The important point is that many of our analysis methods are based on assumptions that are not universally true. There is often a conflict between teaching as much "useful" material as possible and ensuring that the underlying assumptions are covered in detail. One particularly relevant example is the use of elastic methods, and computer programs based on elastic methods, to calculate the internal forces in reinforced concrete structures. This procedure is justified by the safe theorem of plasticity but many engineers today learn elastic analysis without getting a thorough grounding in plastic theory.

In the past structural analysis was not as essential as we consider it today. The gothic cathedrals were constructed without formal calculation, although being based on arches it was possible for their builders to rely on models to ensure stability. The previous discussion of shear shows that in the early days of calculation-based structural engineering most engineers had a very poor understanding of structural mechanics [2]. Despite this they constructed impressive bridges and engine sheds. This was because the profession at the time was pragmatic and usually developed new forms of construction gradually, or carefully using experimental testing. It may be that it the future many engineers will default to accepting computer generated analysis and relying on experience to ensure that the designs are robust. Even in more recent times engineers such as Nervi [10] and Toroja [11] built beautiful elegant shell structures that they justified with simple calculations and model tests. The overall message is that many of our cherished formulae are not universally true and that a pragmatic approach to engineering analysis has worked in the past. A third question that could reasonably be asked is how many engineers need to have truly in-depth knowledge of structural engineering. All structural analysis courses involve compromise.

To oversimplify a little, beam bending theory allows us to calculate the stresses in a floor joist, but scantlings also have their uses (depth of floor joist in inches = half the span in feet +2).

3 WHAT IS ESSENTIAL

The core question of this paper is what is the essential kernel of structural engineering knowledge that civil engineering students should learn in college? Related to this is the question of what civil or structural engineering students should learn in the second cycle, or Masters' years, of a programme if they plan to specialise in structural engineering.

It is tempting to put pen to paper and draw up a list, and ultimately this is what engineering educators do. However, before suggesting some items that should be on the list it is appropriate to consider how engineering practice should influence the list. To see how professional practice influences the topics that are taught it is sufficient to identify methods of structural analysis that have disappeared or are taught with less prominence since the development of digital computers. Graphic statics was one of the most important structural analysis methods before digital computers. It allowed an engineer to use a drawing board to quickly calculate the member forces and support reactions for a statically determinate truss. Looking back, through rose-tinted spectacles, it had the pedagogical advantage of giving real form to the concept of force polygons and made students realize the role of precision in calculation. Few programmes include it nowadays but instead focus on the traditional methods of joint equilibrium and method of sections. However, it may be that in the future these methods will be replaced by the rational approach of writing all the equilibrium equations for a structure and going straight to the solution of these equations as a set. As practicing engineers move away from hand calculations this approach may be preferred.

What of influence line diagrams? These were an important part of structural engineering programmes thirty years ago. They were important because the ability to calculate the worst loading effect for a beam bridge required, or was made simpler by, an understanding of influence line diagrams and Betti's theorem. Nowadays, when the calculation of critical load combinations is usually automated, they are less essential. The flexibility method is also less essential. The flexibility method is now most useful as an easily understood method for the analysis of simple statically indeterminate structures. The previous advantage of flexibility based methods (such as the three-moment equation method) that minimised the number of equations that needed to be solved is no longer significant. In a similar manner some aspects of the stiffness method, such as the shortcuts that could be used when a structure and its loading are symmetric, are no longer important. The relevance of the stiffness method itself is largely important because it is the method that underlies most structural analysis software and basic beam-based direct finite element formulations.

And what of virtual work, the moment-area method, moment distribution, plastic analysis or energy theorems? Taken individually almost every area of structural analysis starts to seem a luxury. The study of plates and shells is hardly core, what about classical elasticity and stress and strain tensors? These may be found in postgraduate programmes but which of them are essential? When some practicing graduate engineers reflect on the traditional analysis techniques they learned in colleges they are sometimes critical or dismissive. This even applies to some research students.

The truth is that much of what is taught at undergraduate level is taught not with a view to structural engineering practice but as a means to developing the students' ability to perform a qualitative analysis with a view to both design and the critiquing of computer-based analyses.

As an initial guess at what is essential for structural engineers today consider the following list:

3.1 *Prepare students to critique solutions*

Students need to develop the skills to critique the output from structural analysis packages. This requires an understanding of axial forces, shear forces, bending moments, stresses, strains and deflections. It is also important for students to understand the assumptions underlying a computer analysis. These are most easily explained in the context of hand-based structural analysis. Therefore hand-calculation based structural analyses of trusses and beams remains relevant.

3.2 *Qualitative analysis and Design*

The ability to interpret the output from a structural analysis program requires good qualitative analysis skills. There is no substitute for a series of lectures tailored to developing qualitative analysis skills, but these skills can be reinforced by the teaching the flexibility method. The flexibility method requires students to use superposition to calculate the deflection of statically determinate structures. This is akin to qualitative analysis with numbers and helps solidify the concepts.

Qualitative analysis also plays a vital part in design. It is important to be able to predict the effect of making changes to a structure.

The interfaces through which engineers access structural analysis software are starting to become so sophisticated that structural engineers can modify the properties of a structure and instantly review the changes in the structural response. It may be that the use of such packages may become an important element of teaching qualitative analysis.

3.3 Teach students the underlying theory and assumptions

A structural analysis programme should ensure that students understand the assumptions that underlie current practice and structural analysis methods based on linear elasticity or more complex methods. First, it is important that students understand that the analyses that they learn are in many cases simplifications and that are not universally applicable.

Accessing the safety of a tie by comparing the tensile stress in the tie with the yield stress of the material is justified if the tie is made from mild steel. Students must understand that if the member is made from a brittle material this assessment would be inappropriate. Even the simplest analyses are based on assumptions that are true in certain circumstances only. Many of our assumptions are material dependent and transmitting this fact is important.

It is important for students to understand when superposition can be assumed and doubly important to recognise when it cannot.

3.4 Postgraduate Programmes

The previous subsections comprise a brief list of essential knowledge that all engineers should have but engineers who undertake further study in structural engineering at Masters level should be exposed to addition material. This is important because this is the means whereby advanced structural engineering topics are introduced into the profession. There is also a need to provide structural engineers who have the basic analytical skills to undertake research and/or to develop the structural analysis software that the profession relies upon.

Interestingly it is not essential that every Masters programme in structural engineering covers every advanced structural engineering concept.

4 A COUNTER EXAMPLE

There are valid reasons for continuing to teach traditional structural analysis methods. As an example, consider the problem of assessing the significance of the additional 2nd order bending moment at the base of a tall structure when it is subject to wind loading. The wind loading causes the tower to bend and in the case of a slender tower the lateral deflection of the tower due to the wind causes additional "P-delta" moments that must be considered. Not all structural analysis software is capable of calculating these effects. It is the nature of current numerical structural analysis packages that they generally yield numerical results without necessarily showing general structural behaviour. In contrast consider the following "old school" analysis, which makes use of virtual work and the moment-area methods.

Figure 3 shows a simple model of a tall slender building subject to a uniform lateral load. Calculating the lateral deflection of the cantilever due to this loading is easily achieving using the moment-area method. The moment in the vertical cantilever is given by the equation

$$m(x) = \frac{\omega L^2}{2} - \omega L x + \frac{\omega x^2}{2} \tag{1}$$

which can be found from static equilibrium. Starting with this equation one can use the moment-area method, which uses the Euler-Bernoulli elastic relationship between moment and curvature, $K = \frac{M}{EI}$, to calculate the lateral deflection of the cantilever, given by Equation 2.

$$y(x) = \frac{1}{EI}\left\{\frac{\omega}{24}x^4 - \frac{\omega L}{6}x^3 + \frac{\omega L^2}{4}x^2\right\} \tag{2}$$

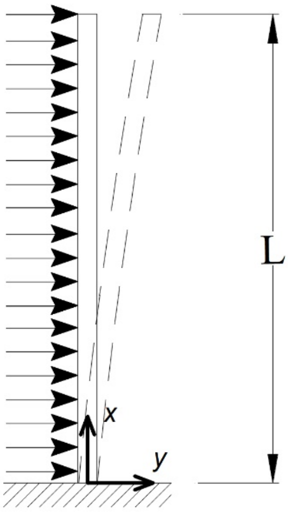

Figure 3. Lateral displace due to wind load.

This gives the deflection of the tower due to the wind but it does not include the P-delta effect, the additional deflections and the additional moments caused by the lateral deflection. However, it is relatively easy to calculate the additional bending moment in the tower due to the lateral deflection due to the wind. The additional moment due to the gravity acting on the deflected cantilever is given by the integral in Equation 3. This equation gives the additional moment at a height of x^* as,

$$m(x_*) = \int_{x_*}^{L} \rho g\big(y(x) - y(x_*)\big)dx \tag{3}$$

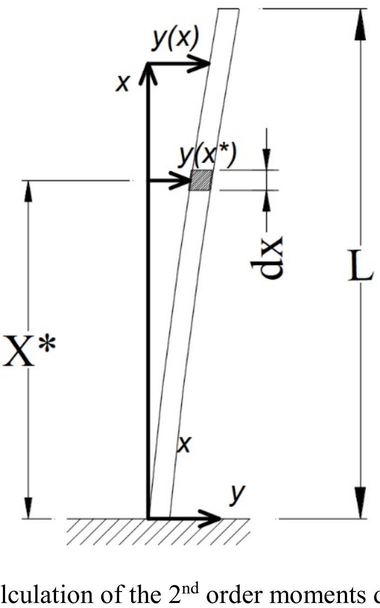

Figure 4. Calculation of the 2nd order moments due to lateral deflection.

Where ρ is the mass of the building per unit height. Performing this integration gives a formula for the addition moment, Equation (4).

$$m_{p\Delta_1}(x_*) = \frac{\omega\rho g}{EI}\left(\frac{L^5}{20}+\frac{x^5}{30}-\frac{Lx^4}{6}+\frac{L^2x^3}{3}-\frac{L^3x^2}{4}\right) \qquad (4)$$

This additional moment, which is shown in Figure 5, in turn gives rise to additional bending in the cantilever and hence additional deflections.

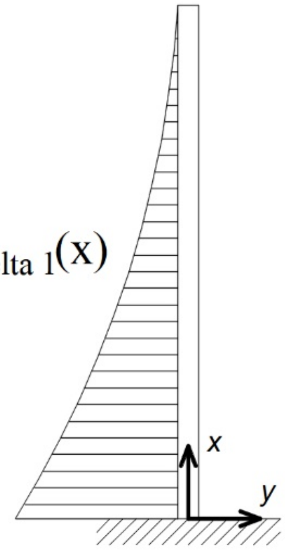

Figure 5. BMD of additional 2nd order moments due to P-delta effect.

The principle of virtual work can be used to calculate the additional lateral displacements due to these moments. Figure 6 shows the bending moments due to a unit lateral load applied to the cantilever at a height of x^*.

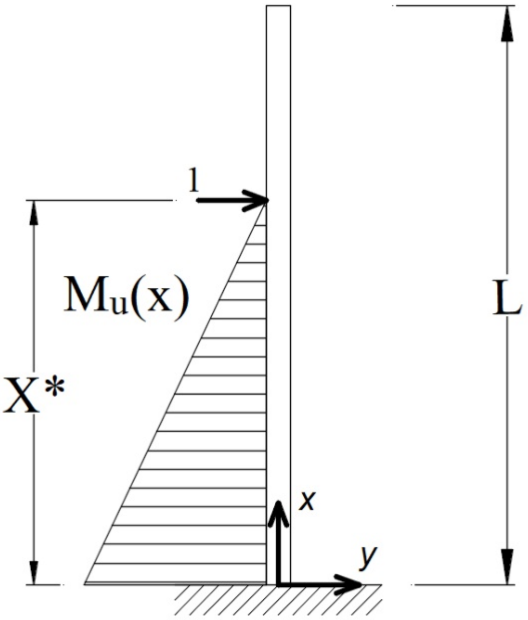

Figure 6. Bending moments in the cantilever due to a unit virtual load applied at a height x^*.

Hence, the displacement at any height x is given by Equation 5.

$$1\cdot\Delta_{x_*} = \int_0^L \frac{m_{p\Delta_1}(x)\cdot m_u(x)}{EI}\,dx = \int_0^{x_*} \frac{m_{p\Delta_1}(x)\cdot m_u(x)}{EI}\,dx \qquad (5)$$

Eventually, with some effort, Equation 6, a formula for the additional lateral displacement due to the additional moments, can be developed.

$$y_{p\Delta_1}(x) = \frac{\omega\rho g}{(EI)^2}\left(\frac{L^5x^2}{40}+\frac{x^7}{1260}-\frac{Lx^6}{180}+\frac{L^2x^5}{60}-\frac{L^3x^4}{48}\right) \qquad (6)$$

This in turn gives rise to a new set of additional moments and a new set of additional displacements, but these can be calculated by repeating the steps shown. In most cases the moment at the base of the cantilever is the critical case and if x is set to zero then the resulting formula, Equation 7 (which includes the first three terms of additional moments), is relatively simple.

$$m(o) = \frac{\omega L^2}{2}\left[1+\frac{1}{10}\left(\frac{\rho g L^3}{EI}\right)+\frac{1}{80}\left(\frac{\rho g L^3}{EI}\right)^2 + \frac{7}{4400}\left(\frac{\rho g L^3}{EI}\right)^3+\cdots\right] \qquad (7)$$

Arriving at Equation 7 requires more than a little effort and reorganisation before it becames clear that the key parameter in this analysis is $\frac{\rho g L^3}{EI}$. The initial term in the equation gives the moment due to wind loading. The sum of the other terms give the additional 2nd order bending moment. The Eurocodes allow designers to ignore the 2nd order effect if it is less than 10% of the moment due to wind loading. Thus is possible to work backwards and identify the maximum value that $\frac{\rho g L^3}{EI}$ can have if the 2nd order moments are to be limited to this or some other limiting value.

The advantage of this old-fashioned analysis is that it has identified the fundamental relationship between ρ, the mass per unit height, L, the height of the building, E, Young's modulus and, I, the second moment of area of the building's plan. This facilitates the initial design of such structures. Of course a good finite element programme with the capability to calculate 2nd order effects can calculate the additional moment for any combination of these variables but won't uncover the underlying relationship.

As it happens this was the result of a real analysis and not a ficticious exercise, therefore there are circumstances when traditional analyses are still justified. However, the question remains as to whether that justifies teaching them to everyone at undergraduate level.

5 FUTURE DEVELOPMENTS

Trying to predict the future is always a challenge and it frequently results in wildly inaccurate predictions. However, the current developments in the application of artificial intelligence and machine learning, when coupled with the recent deployment of BIM make it highly likely that artificial intelligence will be applied in some form to structural engineering design. The data-structures associated with BIM and the ability to incorporate structural engineering models will facilitate the application of machine learning algorithms. Even the more difficult task of scheme design could potentially be tackled by artificially intelligent software, particularly if guided by a structural engineer. The use of packages such as Grasshopper and Rhino, which allow parametric analysis of structures within the Revit BIM package, show how close such an eventuality is potentially. This is potentially good news in that developments such as these will remove much of the tedium of detailed design and will enable designers to consider a much wider range of potential solutions. It is likely that the will make the job of a structural engineer more interesting. On the other hand, it may result in a reduction in the number of structural engineers.

6 CONCLUSIONS

This paper is designed to initiate a discussion of what are the essential skills and essential knowledge that student structural engineers should receive in college. This question arises because of the rapid move from paper-based calculations to digital models, plus the likely developments that will follow as a result of BIM and recent progress in artificial intelligence and machine learning. The author's opening suggestion is that basic hand calculations will be taught to introduce key concepts but that at undergraduate level there will be a greater emphasis on qualitative analysis and an increased emphasis on the stiffness method as a window on finite element analyses. The author suggests that the picture is less clear at postgraduate level because it is important for the profession that a certain number of structural engineers continue to receive tuition in advanced structural analysis topics. It may be that the advent of BIM and machine learning algorithms may lead to the demand for a smaller number of more highly qualified structural engineers.

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