Finite Element Appraisal of the Strut and Tie Method for the Design of Reinforced Concrete Structures

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ABSTRACT: The Strut-and-Tie Method (STM) can be used for the design of reinforced concrete elements such as deep beams and pile caps. STM allows the reduction of complex states of stress, which are known as D- or Discontinuity regions, within a structure to an assembly of simple stress states. Normal beam theory where B- or Bernoulli regions occur, does not apply to these areas. Up until recent times, design for deep beams was generally on simple rules and empirical formulas. However, STM is now included in most modern design code provisions. Yet, there still remains a lack of clear understanding of the STM. In this study, the STM provisions in Eurocode 2, ACI-318 and CAN/CSA-A23.3 are applied to a number of typical elements. These elements are assessed and compared to results from corresponding non-linear finite element analysis using LUSAS finite element software. As experimental testing can be time and cost prohibitive, the ability of the finite element (FE) analysis to appraise the STM is evaluated. From the non-linear FE analysis, a ‘safe' ultimate load is obtained so that excessive cracking is mitigated in line with the limiting deflection criteria in CIRIA Guide 2. The results obtained corresponded well with the STM calculations, with an average factor of safety (FoS) of 1.11. The ultimate failure load from the FE analysis showed a higher average FoS of 1.35. The results showed as the span/depth ratio increased towards a ratio of 3, the FoS decreased. A ratio greater than 3, normal beam theory would apply with corresponding shear reinforcement requirements. This study showed that FE analysis is an effective appraisal method of the STM for elements without transverse (bursting) reinforcement.

KEY WORDS: Strut-and-Tie Method; Reinforced Concrete; Deep Beams; Pile Caps; B- and D- Regions; Finite Element Analysis; Non-Linear Analysis.

1 INTRODUCTION

The Strut-and-Tie Method (STM) is an effective and relatively simple tool in expressing complex stress patterns as triangulated (truss) models i.e. it reduces complex states of stress within a structure to a collection of simple stress paths [1]. STM is generally applied, but not limited, to parts of concrete structures where abrupt changes of geometry occur or near concentrated loads, known as D Regions [2]. Examples of which include pile caps, corbels, beams with holes, connections, deep beams etc., where normal beam theory does not apply. The compression and tension zones within a concrete member are replaced by equivalent struts and ties connected at nodes forming a truss i.e. triangulated model which resists the applied loading [3].

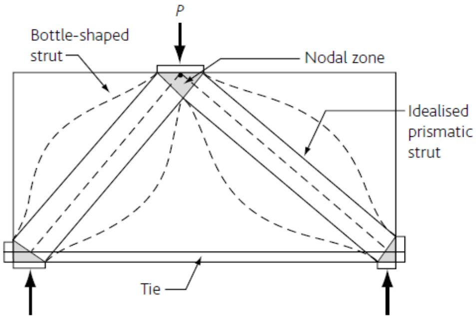


Figure 1. Basic Strut-and-Tie Model components [1].

STM is based on the lower bound theorem of plastic analysis. This means that the distributions of stresses used to resist the applied loading are safe once the following criteria are adhered to [4]: Equilibrium is satisfied through structural elements; The structure is adequately ductile for the proposed struts and ties to develop i.e. the loads to be supported in the manner assumed by the designer [5]; Struts and ties are sized and proportioned to resist the design compressive and tensile forces [1]; The stresses applied to the elements do not exceed their yield or plastic flow capacity [6].

An advantage of strut-and-tie modelling is that a clear load path is provided, and the solution algorithm and equations involved are relatively simple [7]. However, problems arise due to the difficulty in determining the optimum model configuration for the applied loading and this has been subject to ongoing debates [3]. Therefore, the establishment of a clear design process is critical. The design process for strut and tie models is summarised into four main steps [1]: Define and isolate B- and D-Regions; Develop a strut and tie model to represent the complicated flow of forces through the D-region and calculate the member forces within the struts and ties; Design the struts and ties of the model to resist the member forces making sure the members are adequately dimensioned and proportioned; Optimise the model through iteration as necessary to minimise strain energy.

2 STRUT AND TIE MODEL OF ELEMENTS

2.1 *Overview*

Three typical reinforced concrete (RC) elements are presented for analysis. These elements are analysed by the strut and tie method and then further analysed by finite element analysis. The three RC elements are all examples of elements of structure with full discontinuity regions. The strut and tie method is analysed in accordance with Eurocode 2 (EC2) [8], ACI-318-14 (ACI) [9] and CAN/CSA-A23.3-04 (CSA) [10].

2.2 STM Design Flowchart

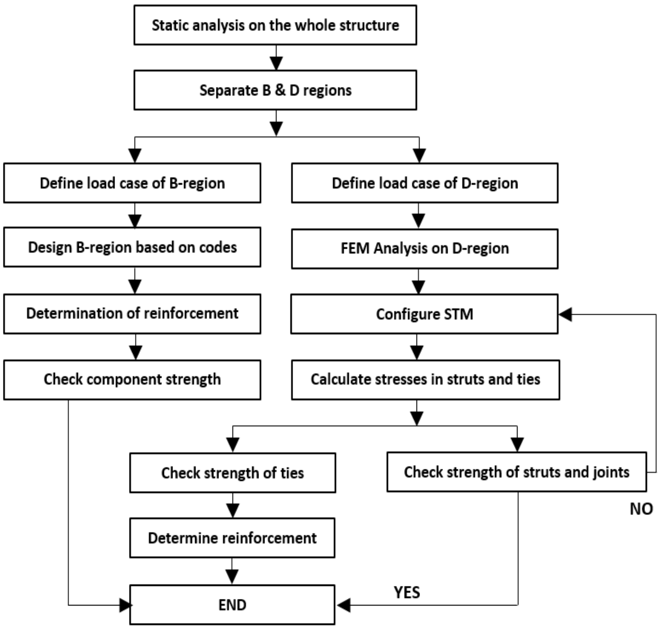


Figure 2. Strut and tie design flowchart [11]

2.3 Strut and Tie Models

The three elements in this study are two deep beams with different b/d ratios and loading conditions (Deep Beam No.1 and Deep Beam No.2 respectively) and a typical pile cap with a point load.

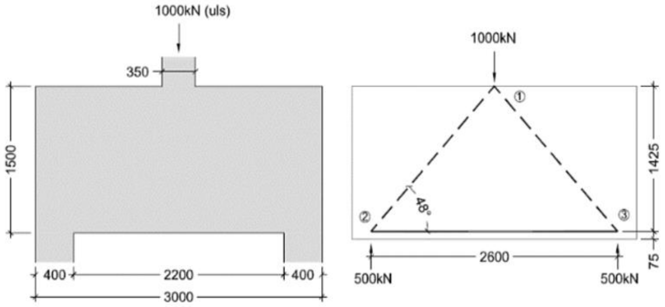


Figure 3. Deep Beam No.1 & Idealised STM

Table 1: Area of Steel Required (Model 1)

Tie	Area of steel required, As_{req} (mm²)		
	EC2	ACI	CSA
2-3	1049	1216	1073

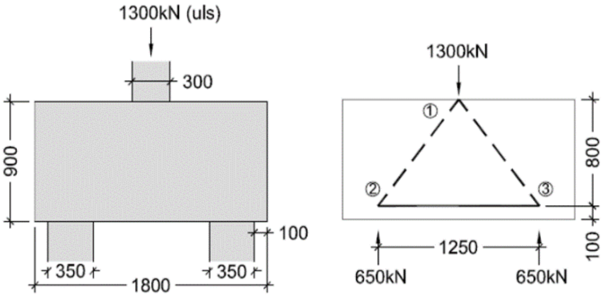


Figure 4: Pile Cap & Idealised STM

Table 2: Area of Steel required (Model 2)

Tie	Area of steel required, As_{req} (mm²)		
	EC2	ACI	CSA
2-3	1168	1354	1195

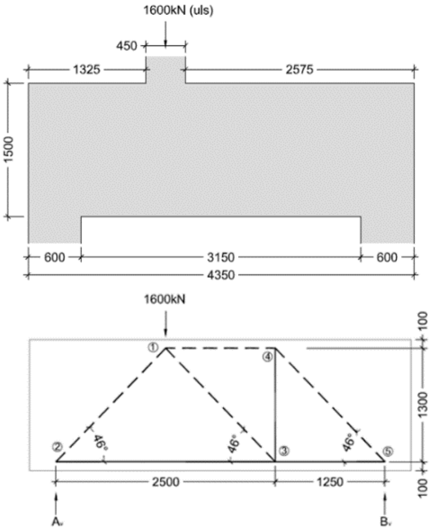


Figure 5: Deep Beam No.2 & Idealised STM

Table 3. Area of Steel required (Model 3)

Tie	Area of steel required, As_{req} (mm²)		
	EC2	ACI	CSA
2-3	2369	2747	2424
2-3	1227	1369	1255
3-5	1227	1369	1255

2.4 STM SUMMARY

ACI 318 is the most conservative approach for calculating the area of steel required. This is due to the applicable partial safety factor applied to the reinforcement. ACI 318 applies a factor of 0.75 in comparison to EC2 with an equivalent factor of 0.87. Therefore, EC2 calculated the least area of steel required, approximately 13.7% less than ACI 318. CSA-A23.3 applies a factor of 0.85 which is close to that of EC2, which is approximately 11.7% less than ACI 318.

When calculating the design resistance of the nodes and struts, EC2 is the most conservative with ACI-318 being the least conservative. The reason for this is related to the factors applied to the concrete compressive strength and the applicable strut and node factors.

EC2 applies a factor, α_{cc}, which is the coefficient taking account of long-term effects on the compressive strength and of unfavourable effects resulting from the way the load is applied. This ranges between 0.8 and 1.0. Goodchild [1], states that α_{cc}, can be conservatively taken as 0.85 for all phenomena. The Irish National Annex gives a value of 0.85 for α_{cc} for flexure and axial loading and 1.0 for other phenomena. If a value of α_{cc}=1.0, is chosen, then the design resistance of the nodes and struts would be close to the values as calculated by the other codes, in between the results of the ACI318 and CSA-A23.3.

One reason for ACI-318 being the least conservative in calculating node and strut design resistances, is that a strength reduction factor of 0.75 is applied to all struts, ties, nodal zones and bearing areas. The other codes define separate factors based on the material i.e. steel or concrete.

3 DEVELOPMENT OF NUMERICAL MODELS

3.1 Development of FE Model

FE Analysis was carried out on LUSAS FE software. Linear elastic models are created first and then checked and validated. After an element study as shown in Figure 6, 2D Plane Stress Continuum Element was selected for the element type. The element shape and function consisted of TPM6 – Quadratic Triangle for the concrete and BAR3 – Quadratic for the reinforcement. Models were optimised through mesh convergence testing. The minimum number of elements is summarised in Table 4. Linear models were then modified for non-linear FE analysis and results obtained and validated with STM.

Table 4. Mesh Convergence

FE Model	Min. Number of elements
Deep Beam No.1	1900
Pile Cap	1000
Deep Beam No.2	2500

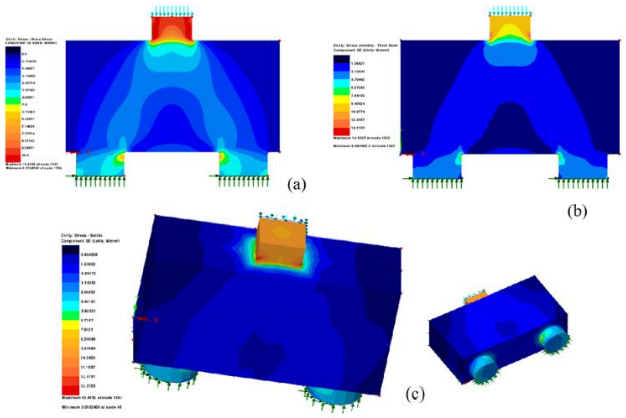


Figure 6. Element Study a) Plane Stress b) Thick Shell c) Solid Continuum

3.2 Non-Linear FEA Properties

The Smoothed Multi Crack Model (Model 109) which is a plastic-damage contact model was selected to model the plastic behavior of the concrete. Applicable properties are provided in Table 5.

Table 5. Concrete Elastic Properties

Element	Concrete Grade	Secant Modulus (GPa)	Poisson's Ratio. v
Deep Beam No.1	C30/37	33.0	0.2
Pile Cap	C28/35	32.3	0.2
Deep Beam No.2	C30/37	33.0	0.2

Table 6. Concrete Plastic Properties

Element	f_{ck}	f_{ctm}	ε_{c1}	ε_{cu1}
Deep Beam No.1	30.0	3.0	0.0022	0.0035
Pile Cap	28.0	2.8	0.0022	0.0035
Deep Beam No.2	30.0	3.0	0.0022	0.0035

f_{ck}: Uniaxial compressive strength
f_{ctm}: Uniaxial tensile strength
ε_{c1}: Strain at peak uniaxial compression
ε_{cu1}: Strain at end of softening curve

The steel (reinforcement) properties are outlined as follows;

Table 7: Non-Linear Steel Properties

Model	Stress Potential	
Young's Modulus	205.0E3	N/mm²
Poisson's Ratio	0.3	
Yield Stress	500	N/mm²
Hardening gradient:		
Slope	2.121E3	
Plastic Strain	1.0	

3.3 Failure Criteria

As already established, the Strut-and-tie method is a plastic analysis. Therefore, it would be extremely conservative to assume the failure point of the model where it stops acting linearly i.e. elastic behaviour, and where the concrete strains do not exceed 0.0035.

To obtain the 'true' ultimate capacity of the FEA model, it can be assumed this occurs at the point of the steel yielding or concrete crushing. However, this would not be a safe design due to excessive cracking in the concrete. The strut and tie model acceptable at Serviceability Limit State (SLS) is deemed adequate for the design of the structure at Ultimate Limit State (ULS). Therefore, there must be some control on the amount of cracking to obtain a safe ultimate capacity and solution to FEA models that should compare well with the lower bound strut and tie calculations.

The CIRIA Guide 2 [12] for the design of deep beams in reinforced concrete, gives guidance on limits to deflection to satisfy excessive cracking. Firstly, it states that the stress in the steel at service load shall not exceed $0.87f_y/\gamma_m$ with the centre-span deflection of a simply supported beam may be presumed to be as follows;

$$\delta_y = \frac{span}{\left[2000\left(\frac{h_a}{l}\right)\right]}, \quad \text{Uniformly distributed load} \qquad (1)$$

$$\delta_y = \frac{span}{\left[2500\left(\frac{h_a}{l}\right)\right]}, \quad \text{Centre Span point load} \qquad (2)$$

These are simplified equations however are very useful in defining the failure point of the FEA models for comparison with the STM calculations. The increase in deflection due to the effect of concrete creep and shrinkage is not likely to be significant [12]. Therefore, it is ignored for this analysis

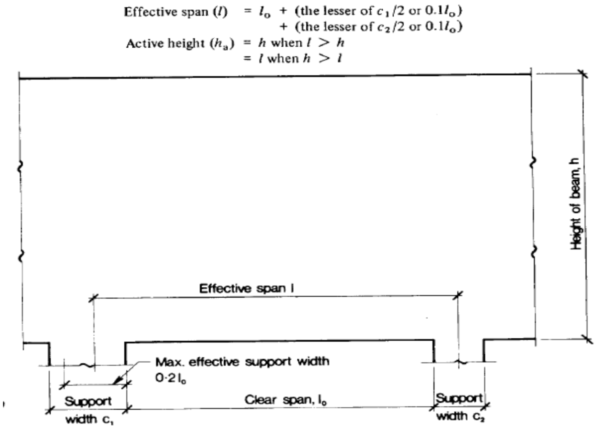


Figure 7. Basic dimensions of deep beams [12]

Table 8. Limiting deflection for FE model

Model	Span	Active height, h_a	Effective Span, l	Allowable Deflection
Deep Beam No.1	2200	1500	2600	1.525
Pile Cap	900	900	1080	0.432
Deep Beam No.2	3150	1500	3750	3.250

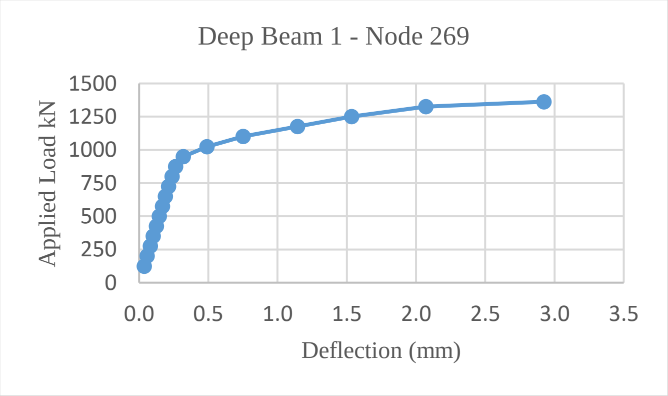
Note: All figures are in millimetres

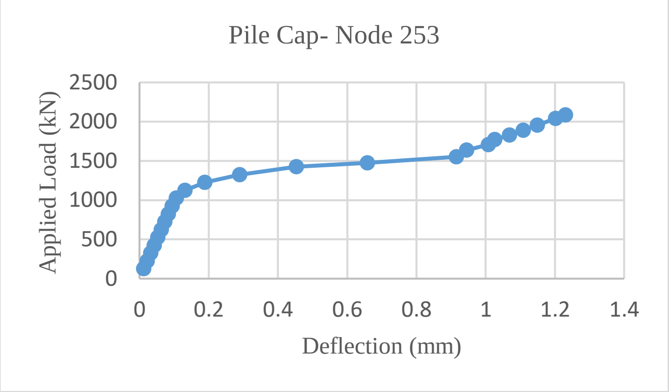
4 NON-LINEAR FEA RESULTS

Once the models successfully converged, the results were processed and analysed.

As per Figure 8, the load-displacement graphs show clearly that the initial displacements are similar to that of a linear model. However, at a certain load (900kN for Model 1, 1000kN for Model 2 & 1100kN for Model 3) the concrete begins to crack and is now in the plastic region. In this region the displacements increase at a higher rate with a smaller increase in applied load compared to the elastic region. The maximum displacement is at the point at which the reinforcement yields which was observed for each model. The FEA results are summarised in Table 9.

4.1 Load Displacement Graphs





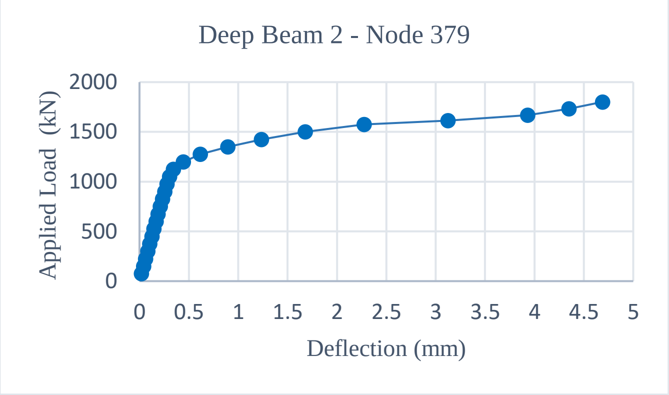
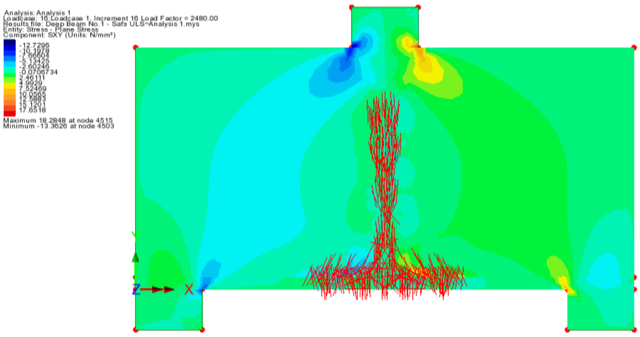


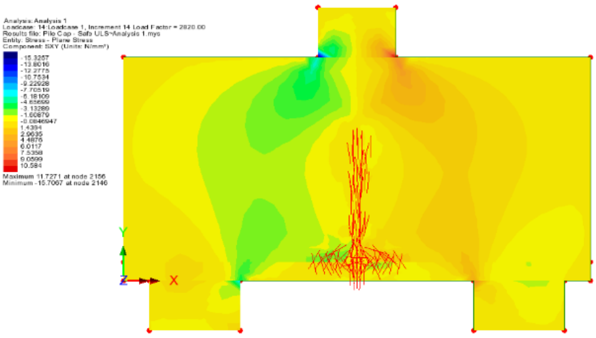
Figure 8. Load-Displacement Graphs

4.2 Crack / Crush Results

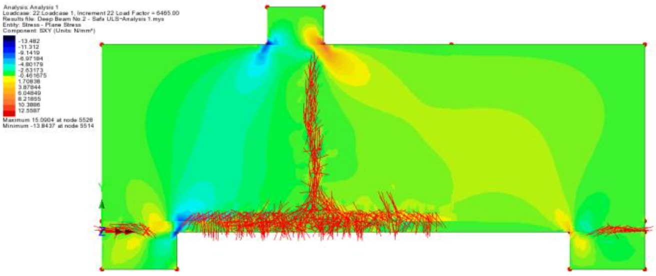
The crack patterns at the 'safe' ultimate applied load obtained from the FE analysis are shown in Figure 9. The crack patterns are illustrated as the red line symbols.



Model 1 - SXY Contour: Applied Load=1240kN 'Safe' Ultimate Load



Model 2: SXY Contour: Applied Load=1410kN 'Safe' Ultimate Load



Model 3: SXY Contour: Applied Load=1616.25kN 'Safe' Ultimate Load

Figure 9. SXY Contour with crack/crush at 'Safe' Load

4.3 *Failure Mode Discussion*

The four main types of failure for deep beams are flexural, shear, bearing and bursting. These failures may be interdependent and occur in combination [12]. From literature, Shuraim [13] observed three of these failures in testing; Shear-compression failure (Bearing), Diagonal splitting (bursting) and diagonal crushing (Shear). On review of the crack patterns (Figure 9) from the FE analysis, each of the models had a similar crack pattern at failure. The cracks generally formed in the centre of the beam at the bottom and propagated up to the top of the beam with increasing load. This would suggest flexural failure. Foster and Gilbert [14] observed in testing that flexural cracks at midspan were first to form and increased to 60-80% of the depth of the beam. Diagonal cracks due to bursting formed suddenly rather than gradually. This also agrees with CIRIA Guide 2, where for flexural failure describes the failure as vertical cracks propagating from the soffit and rise with increasing load to almost the full effective height. It states that failure is due to yielding of the reinforcement and rarely crushing of the concrete. The load deflection curve is typically elastic-plastic.

Therefore, it can be concluded that the FE models are a good representation of reality based on the failure mode and cracking pattern as outlined below:

- Cracking pattern observed for the FE models is similar to that observed for flexural failure in literature;
- Generally, for each FE model, yielding of the reinforcement was first to occur prior to any significant crushing which would indicate flexural failure. The load-deflection curves were also typically elastic-plastic.

5 COMPARISON OF FEA & STM RESULTS

In the STM calculations, the area of steel was calculated based on the applicable design provisions. The area of steel varied in each code due to the applied reduction factor applicable to that code. Therefore, for the appraisal it was decided to analyse the area of steel calculated prior to any factors being applied. That way, the non-linear finite element analysis would not be specific to any code rather the strut-and-tie method in general where a direct comparison can be made. The results of the STM and FE analysis are summarised in the table below.

Table 9. STM-FEA results

Element	l/d	Applied Load		
		STM – Applied Load (kN)	FEA – 'Safe max load' (kN)	FEA – Failure Load (kN)
Deep Beam No.1	1.47	1000	1240	1325
Pile Cap	1.0	1300	1410	2041
Deep Beam No.2	2.5	1600	1616	1801

For Model 1, the 'safe' maximum load is 1240kN as calculated based on the limit on deflection to control excessive cracking. This represents a factor of safety (FoS) of 1.24. The ultimate failure load was also analysed and calculated as 1325kN at the point where the steel yields. This represents a factor of safety (FoS) of 1.34.

For Model 2, the 'safe' maximum load is 1410kN. This represents a factor of safety (FoS) of 1.085. The ultimate failure load was also analysed and calculated as 2041kN at the point where the steel yields. This represents a factor of safety (FoS) of 1.57.

For Model 3, the 'safe' maximum load is 1616kN. This represents a factor of safety (FoS) of 1.01. The ultimate failure load was also analysed and calculated as 1801kN at the point where the steel yields with some signs of concrete crushing. This represents a factor of safety (FoS) of 1.126.

6 DISCUSSION OF RESULTS

As concrete only tolerates limited plastic deformations, a proper and adequate STM based on elastic design shall ensure that the deformation capacity is not exceeded at SLS. As the orientation of struts and ties in the plastic state can deviate from the elastic flow of forces, an incorrect or 'bad' STM would result in excessive crack widths at SLS. This is very important to understand in the evaluation of the results from the FE models.

Firstly, all the FE models predicted a higher 'safe' capacity than the STM results, which is a positive outcome. Panjehpour [4] encountered difficulties in obtaining accurate FEA results as they were calculating well below the STM predicted values.

As shown in Figure 10, Deep Beam No.1 had the largest FoS of 1.24, with the other models much closer to the predicted STM results. The element with the largest l/d ratio, Deep Beam No.2, predicted a FEA 'safe' load nearly identical to that of the STM calculations. This would suggest that the strut-and-tie is a very effective calculation as it is a relatively simple calculation in comparison to the FE analysis. It is also far less time consuming but still predicted the capacity of the elements only an average 11% lower that the FE results. When combined with the different code provisions which include material safety factors etc, the STM is an effective but safe calculation that will avoid excessive cracking and deformations when calculated appropriately.

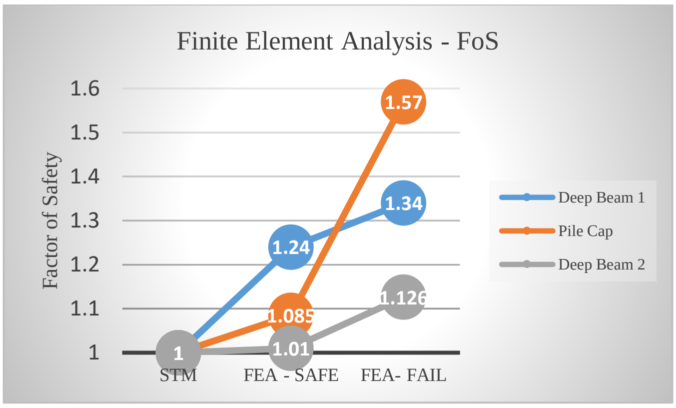


Figure 10. FEA factor of safety on STM results

When assessing the FEA ultimate failure load, there are two vital aspects that assist in appraising the strut and tie method. Firstly, in the STM calculations all three models were calculated so that the unreinforced 'bottle-shaped' strut design resistance was not exceeded i.e. transverse (bursting) reinforcement was not required. Bearing stresses were also not exceeded. This would indicate that the tie (i.e. steel reinforcement) yielding would be the mode of failure.

As discussed, the mode of failure was flexural with the steel yielding in all models prior to any crushing as anticipated by the STM calculations. Secondly, the FE analysis predicted the model with the smallest l/d (i.e. Pile Cap) to have the greatest ultimate failure FoS i.e. at point where the steel started to yield. I.S EN 1992-1-1 2005 states that once the l/d ratio is less than 3 it can be considered a deep beam. As shown in Figure 11, as the l/d ratio increases, the FoS for the ultimate failure load decreases. The graph would suggest as it approaches l/d >3 for 'normal' beam theory, deep beams without transverse reinforcement would have a much smaller FoS, possibly less than 1. This may be due to shear reinforcement i.e. links and minimum reinforcement being required for 'normal' beams in the applicable design provisions.

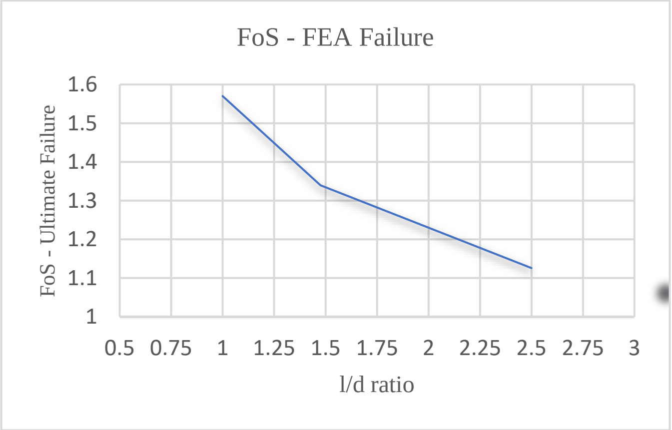


Figure 11. FoS-Fail plotted against l/d ratio

To summarise, the results of the FEA can be deemed sufficiently verified with the comparison of the STM calculations. In turn, it was possible to carry out an adequate FEA appraisal of the strut-and-method as discussed above.

7 CONCLUSIONS

The main findings of the study can be summarized as follows;

- Eurocode 2 is the least conservative in calculating the area of steel required for ties. However, for the design resistance of struts and nodes EC2 is the most conservative., with ACI-318 being the least conservative.
- The LUSAS FE failure crack/crush patterns corresponded well to the description flexural failure and with the crack patterns observed in literature;
- The STM calculations showed that yielding of the steel would occur first which was observed in the non-linear FE analysis models.
- The limiting deflection criteria as per CIRIA Guide 2, was a satisfactory criterion for obtaining the 'safe' ultimate load from the FE models.
- STM is a very effective and relatively simple calculation in comparison to the FE analysis while still obtaining similar results to FEA.
- As the span/depth ratio is increased, the factor of safety of decreases. For a span/depth ratio greater than 3, normal beam theory would apply with shear reinforcement and minimum areas of reinforcement required.

ACKNOWLEDGEMENTS

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