Development and Localisation of Ultra High Performance Concrete using a Particle Packing Model

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ABSTRACT: The research reported here presents the evaluation of two types of UHPC mixes , one using a coarse sand and one using a fine sand, that have been developed for the Irish Construction Industry and to determine if they could be easily replicated using Canadian constituent materials and to illustrate the versatility of the mixes. Canada was selected because of the existing collaboration between the Institute of Technology, Sligo and Fanshawe College, London, Ontario. The mixes were optimised to reduce binder content by using a particle packing model. Optimum particle packing indicated binder contents of 822kg/m^3 and 1022kg/m^3 for the coarse 0/3mm sand and fine 0/0.5mm sand mixes, respectively. In addition to investigating these mixes using equivalent Canadian materials, an additional coarser sand mix using a 0/5mm sand and with a binder content of 667kg/m^3 was investigated. The research illustrates that the mixes developed in Ireland can be readily adapted using Canadian constituents to obtain similar strengths. However, the results also indicate the importance of using particle packing tools to optimise mix designs as the three mixes obtained comparable compressive strengths although the binder content was reduced by 35% from the 0/0.5mm sand mix to the 0/5mm very coarse sand mix. This illustrates how UHPC's can be designed with very low binder contents for some applications to increase their sustainability.

KEY WORDS: Sustainable Concrete, Ultra High Performance Concrete, Concrete Mixture Design, Fibre Reinforced Concrete.

1 INTRODUCTION

As the compressive strength of High Performance Concrete (HPC) mix exceeds the compressive strength of traditional coarse aggregate, the coarse aggregate becomes the weakest constituent in HPC. To further increase the compressive strength of the concrete the coarse aggregate must be removed from the mix. This philosophy has been employed in the development of Ultra High Performance Concrete (UHPC) [1]. In UHPC the coarse aggregate is replaced by stronger ultra-fine particles. These ultra-fine particles also facilitate superior packing of the constituents, which in turn reduces the number of voids within the concrete. UHPC is an encouraging development in concrete technology as it exhibits much higher strength than conventional concrete. As a result thinner structures can be constructed leading to reductions in the structure's self-weight and the volume of concrete and natural raw aggregates used in their construction.

However, a drawback of UHPC is that it results in a significant rise in initial costs over normal and even high performance concretes due to its very high binder contents, which in some cases are greater than 1100kg/m^3, and the use of expensive filler materials such as quartz powder [2]–[4]. Therefore, the cost-efficiency and sustainability of this material must be improved. Currently, in Ireland UHPC is not being used to its full potential. This is mainly because the high cost of producing UHPC is viewed as an inhibitor instead of considering the whole life cycle benefits associated with UHPC [5]. By clearly establishing and demonstrating the advantages of this material, sustainability in the Irish construction industry can be greatly enhanced as demonstrated in studies in other countries [6], [7].

1.1 *Research Significance*

The focus of this study was to evaluate two UHPC mixes, one using a coarse sand and one using a fine sand, which were originally developed for the Irish construction industry and investigate how they might be readily adapted in another country. In the research reported here the Irish derived mixes were modified to use Canadian materials to illustrate the versatility of the mixes and their potential application across different domains. Canada was selected because of the existing collaboration between the Institute of Technology, Sligo and Fanshawe College, London, Ontario. Therefore, it was necessary to use aggregates that are easily accessible in Ireland or Canada for the localised mixes. Constituents that were calculated to be expensive or required a large volume of materials to be imported into the country, and as a result increased the costs and environmental footprint of the UHPC mix, were not considered in the mix design. The mixes were optimised to reduce binder content by using a particle packing model.

2 CONCRETE MIX DESIGN MODELS

There are several tools and methods available for the design of concrete mixes. Methods such as the Linear Packing Density Model (LDPM), Solid Suspension Model (SSM) and Compressive Packing Model (CPM) [8], [9] are unreliable for concretes that have a large portion of fines, making them unsuitable for UHPC mixes due to the high binder contents [10]. An alternative approach is to use integral particle size distribution of continuously graded mixes; thereby allowing very fine particles to be integrated [11]. By using integral particle size distribution a minimal porosity can theoretically be obtained using the optimal particle size distribution (PSD) of all the individually particle materials used in the mix. This approach is known as the Andreasen and Andersen equation [12]. As the Andreasen and Andersen equation does not include a parameter for minimum particle size a modified model was developed to take account of this and is known as the modified Andreasen and Andersen model [13], as shown in equation (1).

$$P(D) = \frac{D^q - D_{min}^q}{D_{max}^q - D_{min}^q} \quad (1)$$

Where $P(D)$ is a fraction of the total solids smaller than size D, D is the particle size (µm), D_{max} is the maximum particle size(µm), D_{min} is the minimum particle size (µm) and q is the distribution modulus. This model has already been successfully applied to the development of several conventional, lightweight and self-compacting concretes [14]. The value of the distribution modulus q determines the proportion between coarse and fines in the mix. By changing this value different types of concrete can be obtained. High values ($q > 0.5$) will lead to a coarse mix and low values ($q < 0.25$) shall result in a mainly fine concrete mix [15]. It has been demonstrated that a q value of 0.0 to 0.28 results in optimal packing of all particles [16] and for self-compacting concretes, such as UHPC, this value should be refined to between 0.22 and 0.25.

3 MIX DESIGN METHODOLOGY

There are currently no national or international standards available that specify the mix design process to achieve a specific UHPFRC strength. Therefore, trial and error tests, and published literature are typically used to develop a new UHPC mix to achieve a specified strength. In addition, in the research reported here the particle size distribution of the individual constituents were also used in conjunction with the modified Andreasen and Andersen model to design the mixes.

Therefore, to develop a sustainable UHPC for a particular country, it is necessary to use constituents that are readily accessible in that particular country. Two types of UHPC's are presented in this research suitable for the Irish or Canadian construction industry, one using a 0/3mm coarse sand and one using a 0/0.5mm fine sand. These mixes highlight how mixes might be adapted between countries and the effect of maximum particle size on the compressive strength of UHPC. An additional coarser mix was also cast in Canada using a 0/5mm coarse sand to further evaluate the effect of using a packing model to develop an ultra-low binder content UHPC.

In addition to the plain UHPC mixes, Ultra High Performance Fibre Reinforced Concrete (UHPFRC) mixes with 2% fibre volume were also investigated to assess the effect of fibres on the strength characteristics of UHPC. Comparable constituent materials were used in both countries to ensure a comparison between compressive strengths achieved by the Irish and Canadian mixes was feasible.

3.1 Constituent Mix Materials

Figure 1 illustrates the particle size distribution of the individual constituents used in each mix in both Canada (CA) and Ireland (IRL).

For the Irish mixes, Rapid hardening Portland cement (RHPC) CEM I Class 42.5R was used as it achieves a higher rate of strength development in comparison to normal cement. In the Canadian mixes, Type HE Cement, which is similar to RHPC CEM I Class 42.5R, was used as it can also achieve a high early age strength. Microsilica was used as the ultra-fine material in both countries as it improves the early age and final strengths, density and durability of the concrete.

In Ireland a fine sand (0/0.5mm) with a PSD in the range of 10µm - 550µm was used for the fine sand mix and a coarse sand (0/3mm) with a PSD in the range of 50µm - 3000µm was used for the coarse sand mix. In Canada, sands with comparable particle size distributions to those used in Ireland were utlised to ensure comparisons could be made. For the fine 0/0.5mm sand the particle size distribution of the material used was 10µm – 600µm instead of 10µm - 550µm used in previous research. For the 0/3mm coarse sand the particle size distribution was 50µm - 3350µm in place of 50µm - 3000µm. For the additional coarser mix designed in Canada, a 0/5mm sand was utilised.

In both Ireland and Canada a polycarboxylate polyer (PCE) superplasticiser with accelerating properties was used as the high range water reducing/superplasticiser admixture. This admixture is specifically used in concrete with high early strength development, high water reductions and excellent flowability. The steel fibres used in all mixes were Dramix OL 13/.20 with a length of 13mm, a diameter of 0.20mm, and a tensile strength of 2600MPa.

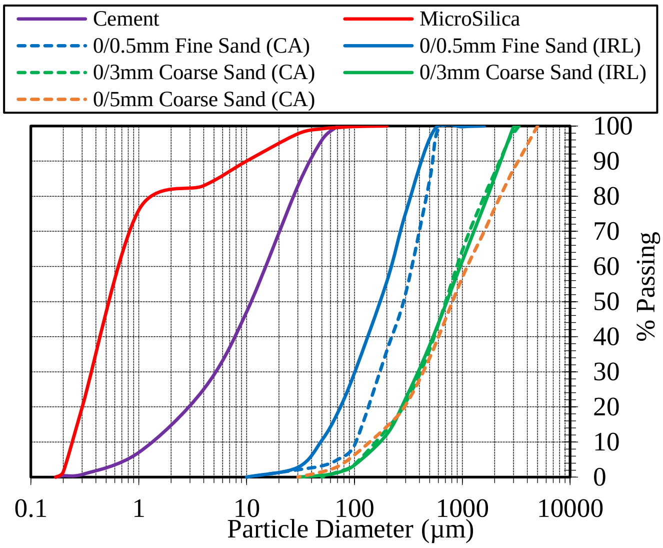

Figure 1. Particle size distributions of the mix constituents.

3.2 UHPC Mix Design

A value of 0.23 and 0.22 was utlised for the distribution modulus q in the modified Andreasen and Andersen model, equation (1), for the coarse sand and fine sand mixes, respectively. A value of 0.23 was selected for the coarse sand mixes as determined from previous literature [10]. This value was reduced to 0.22 for the fine sand mix as the aggregate size is smaller and to maintain q within the recommended limits of 0.22 – 0.25 for self-compacting concrete. The quantities of each mix constituent were optimised to result in the lowest possible least square values when compared to the modified Andreasen and Andersen model target curve.

The final mix design for each plain and fibrous mix is presented in Tables 1 and 2. Each concrete mix was denoted in two parts, the first part was UHPC or UHPFRC to illustrate a plain or 2% fibre mix, respectively. This was followed by a number to illustrate the particle size of the sand utlised in the mix. For example, UHPC-0/3 signifies a 0/3mm coarse sand mix with 0% fibres and UHPFRC-0/0.5 signifies a 0/0.5mm fine sand mix with 2% fibres.

Figures 2 to 4 present the modified Andreasen and Andersen model target curves and the actual mix curves for the UHPC-0/0.5 fine sand, UHPC-0/3 coarse sand and UHPC-0/5 coarser sand mixes, respectively. As the steel fibres are not considered in the particle packing curve the UHPFRC mixes have the same integral grading curve as their UHPC mix counterparts.

Table 1. Material quantities of each plain mix

UHPC Mixes	UHPC-0/5 (kg/m^3)	UHPC-0/3 (kg/m^3)	UHPC-0/0.5 (kg/m^3)
Cement	565	685	810
Microsilica	102	203	203
0/5mm Sand	1495	-	-
0/3mm Sand	-	1317	-
0/0.5mm Sand	-	-	1022
PCE	38	40	42
Water	140	162	178
Steel Fibres	0 (0%)	0 (0%)	0 (0%)
Total Binder	667	822	1013
water/binder	0.21	0.20	0.18

Table 2. Material quantities of each fibrous mix

UHPFRC Mixes	UHPFRC-0/5 (kg/m^3)	UHPFRC-0/3 (kg/m^3)	UHPFRC-0/0.5 (kg/m^3)
Cement	565	685	810
Microsilica	102	203	203
0/5mm Sand	1495	-	-
0/3mm Sand	-	1317	-
0/0.5mm Sand	-	-	1022
PCE	38	40	42
Water	140	162	178
Steel Fibres	155 (2%)	155 (2%)	0 (0%)
Total Binder	667	822	1013
water/binder	0.21	0.20	0.18

As expected Figure 2 demonstrates that as a result of the fine sand used in Canada being slightly coarser than the equivalent sand used in Ireland, there is some deviation of the UHPC-0/0.5 Canadian mix from the target curve in the range of 50µm to 400µm. However, the overall mix curve still exhibits good agreement with the target curve. The similarity between the 0/3mm coarse sands used in both Canada and Ireland are clear from Figure 3, with both UHPC-0/3 mixes exhibiting good agreement with each other and the target curve. The minor discrepancies noted in the range of 10µm to 100µm is associated with the particle space a filler material would typically occupy. As demonstrated in Figure 4 the UHPC-0/5 coarser sand mix developed in Canada exhibited good agreement with the target curve.

3.3 UHPC Mixing and Curing Procedure

The mixing process and time can vary depending on the type and speed of mixer used with typical values ranging from 10 – 25 minutes. Firstly, the sand and silica fume was dry mixed for 3 minutes. The cement was then added and the dry particles were mixed for a further 5 minutes until a uniform dry powder mix was achieved. Over a period of 2 minutes the water and superplasticiser, which were previously mixed together, were added to the dry mix. After a further 4-5 minutes a significant change from a dry to wet consistency of the mix occurred, known as "the turn". After a further 3 minutes a wet paste concrete was achieved. At this point the plain UHPC mixes were ready and specimens were then cast and placed on a vibrating table for compaction. For the UHPFRC mixes, the steel fibres were gradually added to the mix over a period of 1 minute and mixing then continued for a further 3 minutes until a uniform fibre distribution was obtained. Total mixing time was approximately 18 and 22 minutes for the UHPC and UHPFRC mixes, respectively. All specimens were covered with a damp hessian cloth and polythene sheets and kept at a constant temperature of 20°C for 24 hours at which time demoulding occurred. All specimens were then placed in a curing tank at 20°C ± 2°C until testing at 28 days.

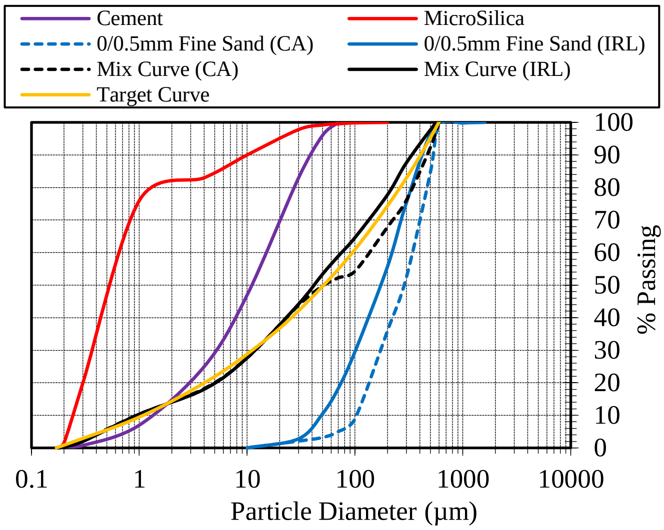

Figure 2. Mix target curve (q=0.22) and the resulting integral grading curve of the composed UHPC-0/0.5 fine sand mixes.

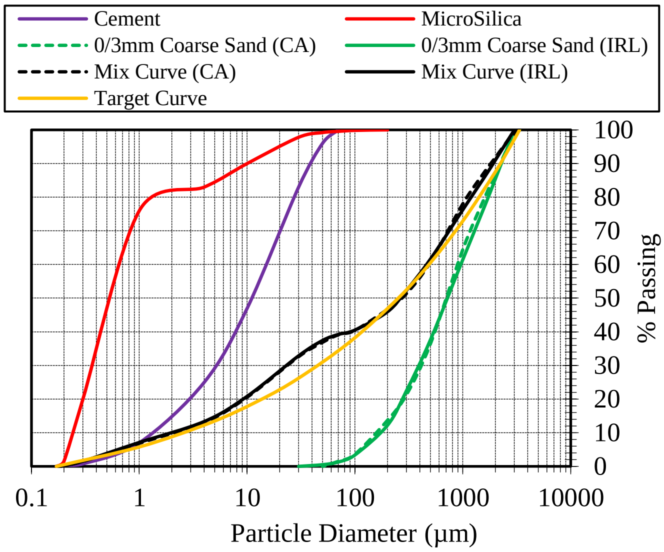

Figure 3. Mix target curve (q=0.23) and the resulting integral grading curve of the composed UHPC-0/3 coarse sand mixes.

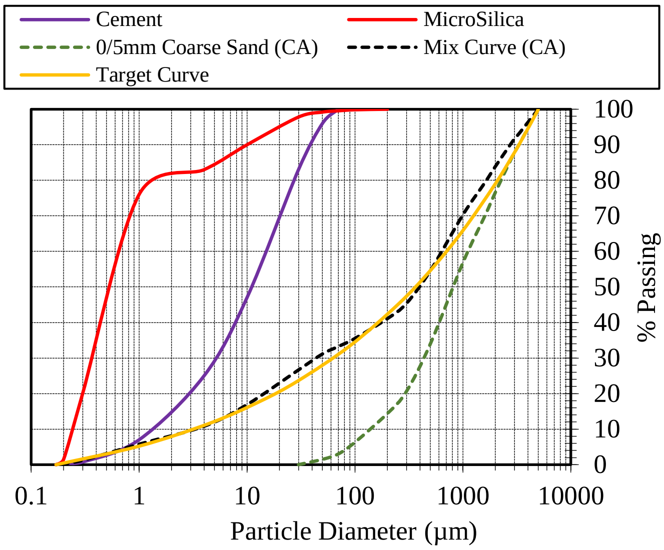

Figure 4. Mix target curve (q=0.23) and the resulting integral grading curve of the composed UHPC-0/5 coarser sand Canadian mix.

4 COMPRESSIVE STRENGTH TESTS

For comparison purposes, compression tests were carried out on 100mm by 200mm high cylinder specimens cast from both the Irish and Canadian mixes. At least three specimens for each mix were tested at 28 days after casting. Careful consideration was taking during casting, striking, curing and testing to eliminate variations in results. In particular, the same procedures for casting and testing were used at all times where possible.

For the Canadian compressive strength cylinder tests load application was controlled using force control at a rate of 0.5MPa/s for the entirety of the test. As the stress-strain behaviour associated with the specimens was also recorded for the Irish compressive strength tests, not reported here, force control loading was not used for the entire duration of the test. Instead loading was switched to displacement-controlled loading at 85% of the expected peak load using three linear variable displacement transducers (LVDT's) equally spaced around the cylinder and attached to the machine and loading continued at a rate of 1μm/s. However, this difference between test procedures would not be a cause for any noteworthy discrepancies between results.

4.1 Compression Test Results

Table 3 presents the average 28-day cylindrical compressive strength results for each UHPC mix developed in Ireland and Canada. It is evident that the UHPC-0/3 and UHPC-0/0.5 mixes developed in Ireland obtained slightly higher strengths than their Canadian counterparts. The UHPC-0/0.5 Irish mix achieved 7MPa higher compressive strength than the Canadian UHPC-0/0.5 mix, which is a result of the Irish mix being more closely aligned with the target curve than its Canadian counterpart, Figure 2. This highlights the importance of using a packing model and obtaining optimum particle packing when designing UHPC and UHPFRC mixes.

Table 3. Cylinder compression strengths

Fibre Content (%)	Mix Type	Irish Mix (MPa)	Canadian Mix (MPa)
0	UHPC-0/5	-	117
	UHPC-0/3	117	115
	UHPC-0/0.5	126	121
2	UHPFRC-0/5	-	133
	UHPFRC-0/3	131	126
	UHPRRC-0/0.5	147	122*

*value indicates testing equipment issue error. Result discarded from discussion but presented here for completeness.

The Irish UHPC and UHPFRC mixes portrayed similar trends to the Canadian mixes with respect to aggregate size. As the total binder content in the UHPC-0/0.5 fine sand mix was 191kg/m^3 higher than the UHPC-0/3 coarse sand mix it was expected that the UHPC-0/0.5mm and UHPFRC-0/0.5mm mixes would obtain higher strengths than the UHPC-0/3 and UHPFRC-0/3 mixes, respectively. However, this strength increase was only 8% for the UHPC mixes and 12% for the UHPFRC mixes. All mixes have very low water/binder ratios, which varied from 0.21 for the 0/5mm coarse sand mix to 0.18 for the 0/0.5mm fine sand mix. This indicates that in all mixes the degree of hydration was relatively small, which is a common attribute of UHPC Physical inspection of the UHPC 0/0.5 fine sand specimens indicated that a large portion of the cementitious material was not hydrated and instead acted as a filler material. This suggests that UHPCs and UHPFRCs can be designed with a lower binder content when good particle packing is achieved, rather than increasing water content to hydrate the excess binder content. Therefore, increasing the strength to cost ratio and reducing the embodied energy to develop UHPFRCs that are more sustainable.

By adding 2% fibres to the Irish UHPC mixes the peak strength of the specimens increased from 117MPa to 131MPa for the 0/3mm coarse sand mix and from 126MPa to 147MPa for the 0/0.5mm fine sand mix. It should be noted that there was an issue with the compression testing rig for the Canadian UHPFRC-0/0.5 fine sand mix and the compression capacity appears to be underestimated. Therefore, this mix is not discussed further here.

The Canadian UHPFRC-0/3 coarse sand and UHPFRC-0/5 coarser sand mixes exhibited increases in strength of 10% and 14%, respectively, over their plain mix counterparts. These values are comparable to the Irish fibrous UHPFRC-0/0.5 fine sand and UHPFRC-0/3 coarse sand mixes that exhibited increases in strength of 17% and 12%, respectively, over their plain mix counterparts. Therefore, it appears that in compression the expected increase in compressive strength from a UHPC mix to a 2% micro-steel fibre mix will typically range from 10% to 17%, which is a moderately low increase in strength in comparison to the significant increase in cost and environmental impact due to the addition of fibres.

5 TENSILE STRENGTH RESULTS

Due to equipment constraints in the Canadian laboratory, only the Irish specimens were tested under in-direct tension using the standard prism flexure test with four-point loading.

However, due to the similarity of both the mix designs and the compression test results, the in-direct tension results for the Irish mixes should correlate well with the Canadian mixes. At least three specimens for each mix were tested at 28 days after casting. Specimens had a span of 300mm and a breath and height of 100mm. The purpose of this test was not just to determine peak tension stress but to also evaluate the residual strength given by the fibres after first cracking occurred. The test was initially conducted at a speed of 2µm/s with servo feedback from the average of two deflection LVDT's placed on either side of the specimen at midspan. The test speed was increased to 8µm/s when peak strength was reached.

5.1 *Tensile Test Results*

Table 4 presents the average 28-day beam flexural strength for each mix. Similar to the compression results for the plain mixes, in flexure the UHPC-0/0.5 fine sand mix only exhibited a minor increase of strength of 7%, from 14.1MPa to 15.1MPa, over the UHPC-0/3 coarse sand mix. The first crack stress of the UHPFRC mixes were slightly higher than the UHPC mixes in both cases, this demonstrates that fibres have an insignificant effect on the pre-first crack of UHPFRC. If the peak strengths of the UHPFRC mixes are considered, the UHPFRC-0/3 coarse sand mix demonstrated a strength increase of 33%, from 14.1MPa to 18.7MPa and the UHPFRC-0/0.5 fine sand mix demonstrated a significantly higher strength increase of 75%, from 15.1MPa to 26.5MPa over their plain mix counterparts. This illustrates that the inclusion of steel fibres in UHPC has a substantially higher impact on the flexural strength than on compressive strength. Due to the self-compacting and flowability nature of UHPC, as the mix is placed in the moulds it flows so that the steel fibres align themselves with the direction of the flow which is parallel to the flexural stresses in the beams. As the fibres bridge micro and macro cracks, both the pre-cracking and post-cracking strength and behaviour of the concrete is enhanced. It was noted in the UHPFRC prism specimens that the fine sand mix had a higher percentage of fibres aligned with the direction of the flow in comparison to the coarse sand mix. This illustrates why the fine sand fibre reinforced mix exhibited a higher percentage increase in strength. It can be concluded that the smaller particle size in the fine sand mix encouraged better alignment of the fibres with the direction of flow when casting the specimens. This suggests that by giving careful consideration to how UHPFRC is placed in structural members, such as beams and slabs, the fibres can be aligned to give maximum performance and strength.

Table 4. Irish mix beam flexural strengths

Fibre Content (%)	Mix Type	First Crack (MPa)	Peak Strength (MPa)
0	UHPC-0/3	14.1	14.1
	UHPC-0/0.5	15.1	15.1
2	UHPFRC-0/3	16.4	18.7
	UHPFRC-0/0.5	15.7	26.5

6 SUSTAINABILITY ANALYSIS

To assess the sustainability performance of the UHPC and UHPFRC mixes, an analysis was conducted to determine the binder efficiency of the mixes. Binder efficiency is a measure of the total amount of binder to strength ratio and is used to determine the amount of binder required to produce 1MPa of strength. Therefore, the lower the binder efficiency the more efficient the mix is considered to be, as it requires a lower binder content to obtain the required strength. The binder in UHPC typically has both the highest cost and embodied energy of the constituent materials and can therefore be used as a simplified method to measure the sustainability of a particular concrete mix design.

Figure 5 presents the binder efficiency of the mixes in terms of cylindrical compressive strength. The results indicate that for both the 0% and 2% fibre contents as the sand size increased the binder efficiency also increased. This is a result of the binder content reducing as aggregate size increased, see Table 3, to ensure optimal particle packing was achieved. With the inclusion of 2% fibres there was only a minor increase in the binder efficiency of all mixes. This is a result of steel fibres only having a moderately small effect on the pre-cracking and peak compressive strengths of UHPFRC. An analysis of typical UHPFRC 2% steel fibre mixes in the literature indicated that the binder efficiency range in UHPFRC is typically between 6.2kg/MPa to 9.3kg/MPa with an average value of 8.1kg/MPa [4], [10], [17]–[19]. Therefore, this research illustrates how sustainable UHPFRC mixes can be developed without the need to use mix designs with constituents comparable to those of UHPFRCs currently available in other countries.

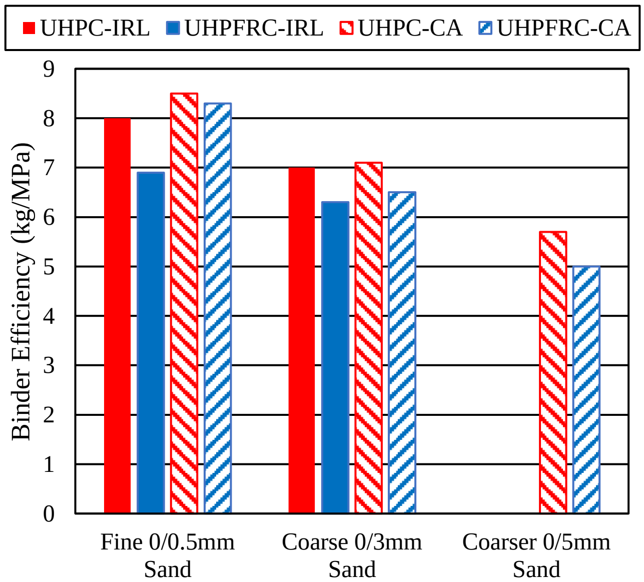

Figure 5. Binder efficiency of the mixes in compression.

Figure 6 presents the binder efficiency of the Irish mixes with respect to tensile performance. As a result of the majority of the fibres aligning with the direction of flow in the UHPFRC-0/0.5 fine sand mix the binder efficiency is almost twice that of its plain mix counterpart. As the fibres in the UHPFRC-0/3 coarse sand mix did not align with the direction of flow to the same extent as the fine sand mix the binder efficiency improvement in the fibrous coarse sand mix in comparison to its plain mix counterpart was not as significant.

When both the compression and flexural strengths are considered it appears that the most sustainably advantageous mix developed is the fine sand UHPFRC mix with 2% fibres due to its enhanced flexural performance.

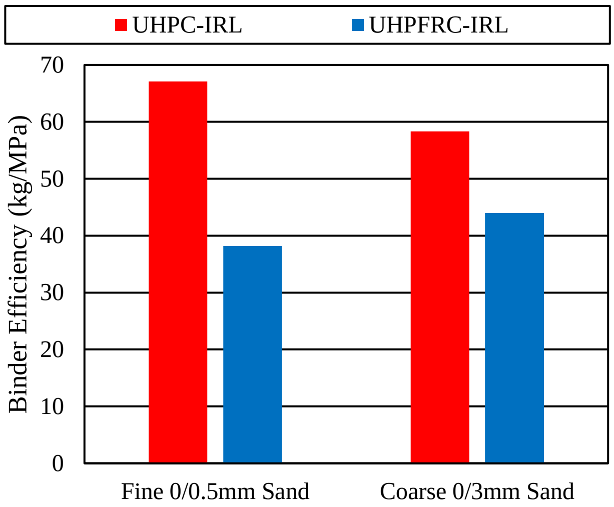

Figure 6. Binder efficiency of the mixes in tension

Further research is required to investigate the performance of the Canadian mixes in tension. Particularly, as the performance of the 0/5mm sand mix with 2% fibres exhibits excellent compressive strength results due to its low cost and embodied energy in comparison to typical UHPFRCs.

7 CONCLUSIONS

This paper presents the findings of a new UHPFRC mix designs that are made more sustainably viable in terms of strength, cost and environmental impact by reducing or removing the inclusion of filler materials such as quartz flour. The following conclusions can be drawn based on the results:

- It is possible to design UHPC mixes using the Andresen & Anderersen model based on the maximum and minimum particle size used, excluding fibre size.
- By using a fibre volume of 2% the compressive strength of the UHPCs increased by only 10 to 17%, which illustrates that the steel fibres have a relatively small effect on the pre-cracking and peak compressive strengths of UHPFRC.
- By substituting equivalent Canadian constituents into the mixes developed in Ireland similar compressive strengths were achieved, illustrating the versatility of the proposed mixes and the potential for localisation of UHPC mix designs using comparable materials.
- It is possible to produce a UHPC with a cylindrical compressive strength of greater than 110MPa with a total binder content of less than 700kg/m^3.
- The flexural strength of the UHPFRC 0/3mm coarse sand and 0/0.5mm fine sand mixes increased by 33% and 75%, respectively, with the addition of a 2% steel fibre volume. This illustrates the potential advantages of using steel fibres in UHPFRC for structural members.
- With 2% fibres the 0/0.5mm fine sand mix has both the highest cost and environmental impact due to the high binder content and fibres. However, due to its superior physical and mechanical characteristics, in particular flexural strength, it is likely to be the most beneficial mix over a structure's life cycle.

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