Simulation of Traffic Loading on Long Span Bridges

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ABSTRACT: Existing long span load models have typically been developed using a number of conservative assumptions, and as such are more applicable to the design of new bridges rather than the assessment of existing structures. Excessive conservatism in such assumptions can lead to expensive and unnecessary interventions in existing bridges. Furthermore, existing load models do not always allow for correlations in traffic weights and vehicle positions on the bridge. This paper introduces a method of simulating the load effect on long-span bridges, termed 'Long Span Scenario Modelling' (LSSM). The scenarios are blocks of vehicles extracted from a congested traffic stream that contain the inherent correlations between vehicle weights and positions. For long-span bridges, the combination of vehicles, such as platoons of Heavy Goods Vehicles (HGVs), has a greater influence on bridge loading than individual heavy vehicles. The scenario modelling approach allows for these critical vehicle combinations. Weigh-in-Motion (WIM) data from a site in the USA is used to demonstrate the process. The LSSM is shown to better represent the long span load effect when compared to measured traffic, particularly when the correlation between successive scenarios is accounted for.

KEY WORDS: Bridge; Loading; Long-span; WIM.

1 INTRODUCTION

1.1 Motivation

Traffic loading for long-span bridges is generally not addressed in codes of practice and extending load models from shorter spans can lead to simplified and conservative assumptions. Unlike short to medium span bridges where individual heavy vehicles in free flowing traffic produce the maximum load effect, congested traffic events give rise to the critical loading events for long span structures [1-3].

Driving behaviour in free flowing traffic can generally be replicated using models such as the Poisson arrival process, normalised headway model or the headway distribution statistics model [4-6]. As driving behaviour significantly influences congested traffic, its modelling is more complex.

Recent studies have used micro-simulation to represent traffic behaviour in congested conditions on long-span bridges. Cellular automata, where the bridge is divided into cells considers lane changing behaviour, but not the variability of vehicle lengths and gaps [7]. Car following models combined with lane changing models have been used to determine long span loading effects under a variety of congestion types [8-13]. Importantly these models have highlighted that the critical conditions for long span loading is not always the widely-used full-stop condition, but can be slow moving traffic [11, 14]. This can occur because full-stop queues consider only one realisation of vehicles on the bridge compared to recurring congestion where the frequency of occurrence is much higher.

A drawback of mico-simulation however is that there is limited data available to calibrate the vehicle models [15, 16], and despite recent advances [12] it can be computationally intense. Combining a gap model with a suitable arrival process with has advantages in that it is computationally efficient and can more readily used for assessment of long span bridges by practitioners [17].

Early developments of codes of practice in the USA and the UK based their traffic load models on alternating heavy and light vehicles at spacings of 9.1 and 16.8 m respectively to represent slow moving jams [18, 19]. For fully stopped traffic, based on limited measurements, FNP [3] proposed a truncated normal distribution of vehicle gaps for loaded lengths between 100 and 1000 m, with constant values of 1.35 and 2.7 m for shorter and longer lengths respectively. The development of a recent long span load model in Korea combines a WIM database with a number of deterministic traffic congestion 'scenarios', where the axle to axle gaps are reduced to constant value of 4.5 m. The underlying traffic model for the Eurocode was developed based on simulations of congested traffic moving at slow speeds (5-10 km/h) and fully stopped traffic with constant axle to axle gaps of 5 m [20]. The update of the AASHTO load model to account for long span bridges simulated a series traffic jams with a constant axle to axle gaps of 7.6m [21]. In terms of slow moving traffic, Buckland et al. [22] took account of speed-space relationships based on observed videos, while Vrouwenvelder and Waarts [23] proposed random axle to axle gaps of 4-10 m depending on vehicle speed. Bailey [4] develops notable models that represent gaps as statistical distributions for both full stop [24] and congested traffic conditions [25]. The latter model accounts for a range of vehicles speeds from 18-72 km/h.

A variety of vehicle arrival processes have also been used by researchers: vehicle patterns from free flowing traffic have been maintained but inter vehicle gaps reduced in a number of studies [1, 17, 26], while they have been randomly generated in others [23, 27]. Crespo-Minguillon and Casas [6] used a Markovian vehicle arrival process based on a transition matrix computed from real traffic. A number of studies have also excluded cars from the traffic stream [1, 20, 28].

Enright et al. [29] propose a model for long span bridge loading to replicate the platooning behaviour of cars and trucks. Cars are allowed to change lanes, with the probability of lane changing increasing as the number of trucks it follows increases. Increases in total load of up to 25 % are shown to be possible because of the platoons that form due to lane changing.

A novel scenario modelling approach was developed by OBrien and Enright [30] for short and medium span bridges where patterns of HGVs were sampled from free flowing traffic and used to simulate longer periods of bridge loading. The method was found to capture the correlations present in the measured traffic and was validated using mico-simulation [31].

This paper proposes a method of simulating load effects on long span bridges termed 'Long Span Scenario Modelling' (LSSM). The 'scenarios' are blocks of vehicles extracted from a stream which contain the inherent correlations between vehicle weights and positions. The correlation in load intensity between successive scenarios is explicitly modelled. The scenarios can be used to simulate congested conditions for the required number of congestion events. A large WIM dataset from a site in the USA is used to demonstrate the process. Free-flowing WIM data is converted into a congested traffic stream using lane changing and gap distribution models. Recurring rush hour type congestion is simulated. The total load effect for a 1000 m loaded length is determined for 75- and 1000-year return periods.

2 LONG SPAN SCENARIO MODELLING

2.1 *Methodology*

A significant number of parameters are required to fully describe a congested traffic stream. The number increases with the length of the stream as they include Gross Vehicle Weight (GVW), in-lane and adjacent lane positioning of each vehicle. A significant advantage of extracting scenarios from measured traffic data is that the scenarios automatically contain the vehicle parameters and their correlations [32].

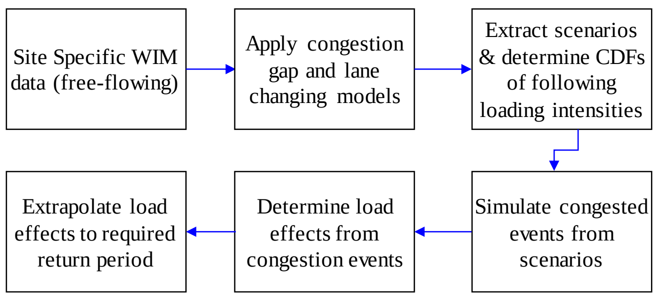

Figure 1 Long Span Scenario Modelling (LSSM) process

The proposed 'Long Span Scenario Modelling' (LSSM) process is illustrated in Figure 1. The process takes site specific WIM data and determines load effects for long span bridges for a given return period. The WIM data, generally collected under free-flow conditions, is first modified to simulate a congested traffic stream from which 'scenarios', typically of length 150 m, are then extracted. Correlations between parameters within the sampled length are implicitly included in each scenario, however correlation between successive scenarios needs to be explicitly modelled. The extracted scenarios are used to simulate congested conditions for the required number of congestion events. The maximum load effect is calculated for each event and the results extrapolated for the required return period.

2.2 *Free-flow to Congested Conditions*

WIM data is typically collected under free flowing traffic conditions, with many WIM technologies unable to collect reliable data during congestion due to the acceleration and deceleration of passing vehicles [33]. To simulate congested conditions it is important to allow for the increase in lane density and for the tendency of vehicles to change lane.

As vehicles approach the congestion location, lane density increases as the gaps between vehicles decrease. Rather than adopting a single value, beta-distributed bumper to bumper gaps are considered which allow for varying gap distributions for different traffic speeds (Figure 2) [4].

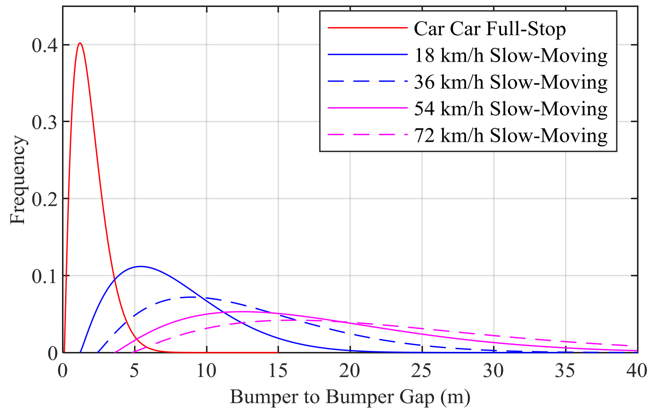

Figure 2 Beta distributions of bumper to bumper vehicles gaps for slow moving [34] and fully-stopped traffic [24]

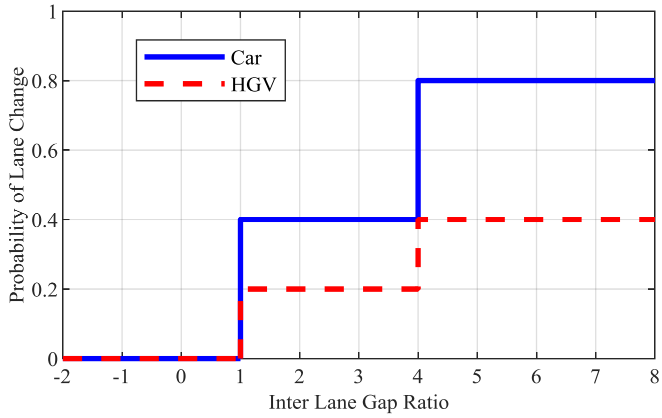

Figure 3 Lane changing model

The vehicle arrival sequence is taken directly from the free-flowing WIM dataset. As vehicles may change lane as traffic becomes congested, a set of the lane changing criteria is used to redistribute lane assignments from the free-flowing to congested conditions [26]. The model assigns a probability of lane changing based on the 'inter-lane gap ratio', defined as the difference in length between the queue in the lane of travel and the adjacent lane divided by the length of the approaching vehicle (Figure 3). A lower probability of lane changing is assigned to trucks based on previous studies [35]. The lane changing process results in a greater number of truck platoons forming, which is significant for long-span bridge traffic load effects [29]. Load effects have been shown to be insensitive to the exact values of the lane changing parameters chosen, but sensitive to the adoption of a lane changing model.

2.3 Scenario Extraction

The next step of the LSSM process is to extract the traffic scenarios by segmenting the congested traffic stream. A scenario length of 150 m has been chosen as it allows key traffic patterns to be captured, i.e. typically up to 6 HGVs in a convoy, while allowing for a significant number of scenarios to be captured. This can be important during night-time periods when low traffic flows limit the length of the congested traffic stream.

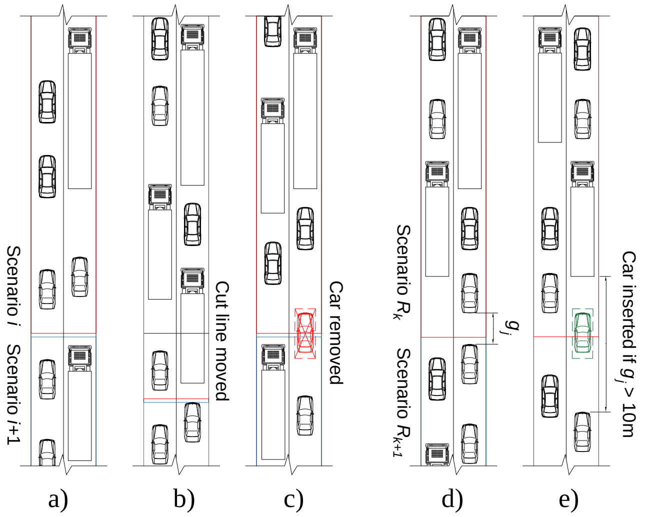

Figure 4 a-c) Extraction of scenarios from congested traffic streams; d-e) Assembly of scenarios

A sample segmentation is illustrated in Figure 4a for a two lane carriageway. This is the simplest case where the scenario cut line falls within the inter-vehicle gaps in each lane. If the cut line coincides with a HGV in either lane, the scenario length is extended to the front of the following vehicle (Figure 4b). In cases where the cut line coincides with a car, the scenario is created but the car is removed (Figure 4c). This facilitates ease of joining scenarios when congestion events are simulated. Extracted scenarios are sorted into bins relating to the hour of the day in which they occurred.

2.4 Correlation between Scenarios

While the extracted scenarios automatically contain the vehicle parameters and their correlations, correlation between the load intensities of successive scenarios needs to be explicitly studied and modelled, particularly when the ratio of loaded length to scenario length is high. The load intensity is defined as the sum of vehicle GVWs in the scenario divided by the scenario length.

Figure 5 illustrates the Pearson coefficient of linear correlation for the load intensities of successive 150 m scenarios for a sample size of 16,549 scenarios. The coefficient is a measure of the linear correlation between two variables where 0 indicates no correlation and 1 indicates a positive correlation [36]. It is evident that there is a positive correlation in the load intensities of successive scenarios, particularly for heavier scenarios which are critical for long span load effects.

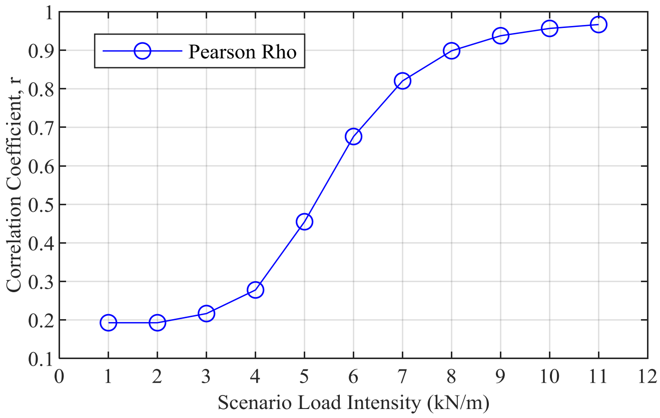

Figure 5 Correlation of load intensities of successive scenarios

The section of a measured congested traffic stream which generated the critical load effect for a loaded length of 1000 m on a sample day is illustrated in Figure 6. The GVW of each vehicle is plotted against its length. The segmented scenarios are marked by differing colours. This critical load combination, particular in Lane 1 (slow lane) is a platoon of moderately loaded HGVs with cars interspersed. The correlation between load intensities of successive scenarios is apparent and it is therefore important that any proposed simulation method allows for such critical vehicle combinations to be present.

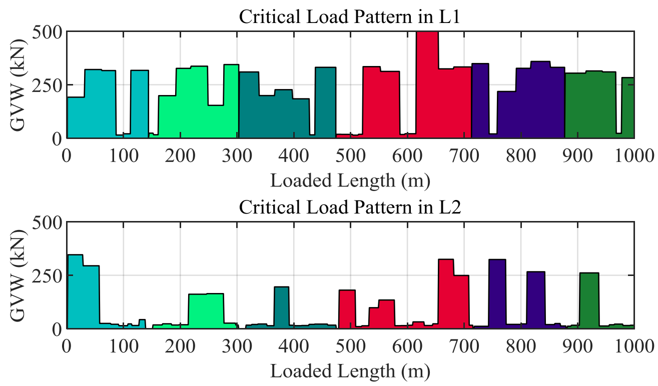

Figure 6 Details of traffic stream generating critical load effect for a 1000 m loaded length

2.5 Correlation Modelling

To allow for the correlation between successive scenarios, scenarios are subsorted into bins corresponding to their load intensity. The chosen bin width is generally 2 kN/m. A bin width of 4 kN/m is adopted for the 14-18 kN/m range due to the small number of scenarios with a load intensity greater than 16 kN/m. For each leading scenario, *i*, the load intensity of the following scenario, *i*+1, is recorded. In this example the load intensity value represents the sum of loading from Lane 1 and Lane 2.

The histogram of following load intensities for the bin of 14-18 kN/m scenarios is illustrated in Figure 7. A number of functions were fitted to the data, including normal, general extreme value, Weibull and gamma distributions. In order to quantify the goodness of fit for each distribution, a least square measure was applied [37]. The gamma distribution produced the best fit to the measured data and was therefore adopted.

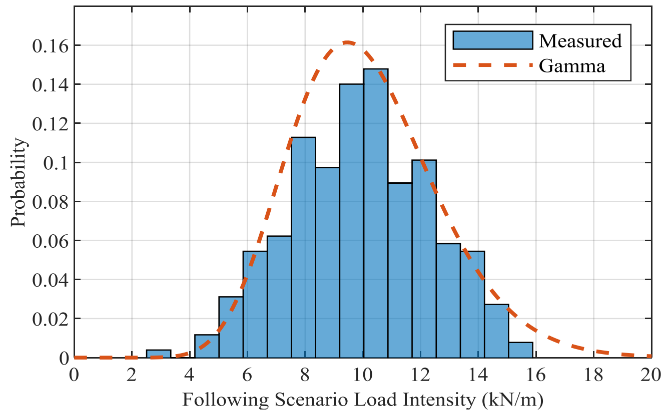

Figure 7 Histogram of following scenario load intensities for scenario bin of 14-18 kN/m

Cumulative Distribution Functions (CDFs) for the load intensities of the following scenarios for each bin are illustrated in Figure 8. The correlation between the load intensities of successive scenarios is again evident, particularly for the heavier intensities. For example, the cumulative probability that the following load intensity is less than 10 kN/m is 0.87 for the 2-4 kN/m bin, but is only 0.54 for the 12-14 kN/m bin.

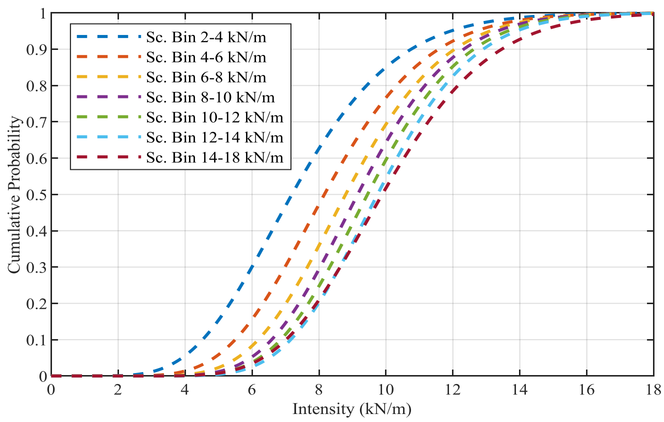

Figure 8 CDFs of following scenario load intensities

2.6 *Congestion Simulation*

To simulate a congestion event using the LSSM process, the hour of occurrence and duration of the congestion event must be selected. These are site specific values which relate to the traffic conditions at that location.

The first scenario is randomly selected from the relevant hour bin. A random number between 0-1 is selected and a value for the load intensity of the following scenario is determined from the relevant CDF (Figure 8). A scenario corresponding to closest value of this load intensity is then selected from the hour bin. The above process is repeated until the required length of congested traffic stream is generated.

In cases where the gap between vehicles in successive scenarios, g_j, is greater than 10 m for slow moving congestion events, a car is inserted into the space (Figure 4e). While the weight of a car is small compared to that of a HGV for the purpose of long span load effects, the insertion offsets the removal of cars in the extraction process and ensures a consistent traffic stream.

3 APPLICATION OF PROPOSED PROCESS

3.1 *Traffic Data*

To illustrate the application of the LSSM process WIM data from both lanes of the westbound I-40 in Tennessee, USA is used. The data, from 2nd January to 31st December 2008, was collected as part of the Federal Highway Administration's (FHWA) Long-Term Pavement Performance (LTPP) program [38]. Data from weekends and public holidays was removed and the data processed to remove any erroneous records in line with established guidance [39, 40]. Approximately 2.6% of the records were removed, resulting in a dataset of approximately 4 million vehicles.

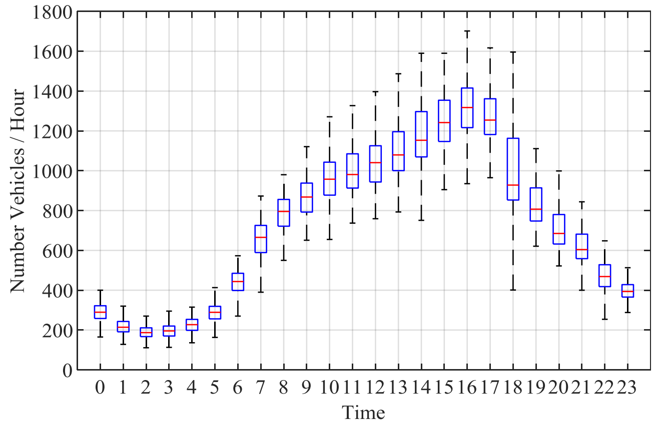

Figure 9 Boxplot of I-40 westbound hourly traffic flows

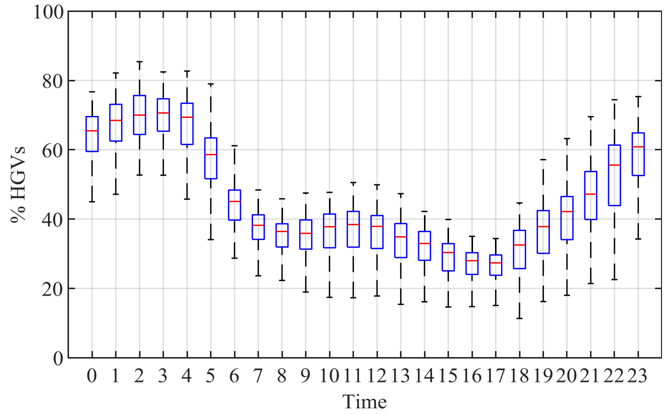

Figure 10 Percentage of HGVs in I-40 traffic flows

The site had a moderate Annual Average Daily Traffic (AADT) flow of 34,862 vehicles in 2008, with an average of 5,251 HGVs per weekday in the westbound direction. Figure 9 and Figure 10 illustrate the variation in traffic flow and percentage of HGVs throughout the day (for both lanes). While the percentage of HGVs is very high during night-time periods, this corresponds to periods of low traffic flow.

3.2 *Congestion Parameters*

At this site, the period of highest flow is between 16-17.00 with a median flow of approximately 1300 vehicles per hour per two lanes and a median percentage of HGVs of 28%. The site is representative of a rural location with moderate traffic flow.

For the purposes of this example the WIM data will be used to simulate daily recurring 'daily rush hour' type congestion to determine the 75- and 1000-year load intensities for a notional long span structure. The congestion is assumed to have a constant speed of 18 km/h with the relevant gap curve selected from Figure 2. The mode of this distribution is approximately 5.4 m, compared to a mode of approximately 1.2 m for fully stopped conditions [24]. The duration of the congestion is assumed to be 1 hour, with congestion occurring each workday. In total 250 working days are simulated which corresponds to one year.

It is to be noted that the percentage of HGVs in the traffic flow is known to decrease with increasing traffic flow as professional drivers plan their driving patterns to avoid regular delays [41]. The percentage HGVs in the traffic flow at this site would be expected to drop to below 15% for congested conditions [10], however for the purpose of this example no changes have been made to the WIM dataset.

3.3 Bridge Type

A 1000 m loaded length is assumed for this work. The studied load effect is 'total load' which is representative of long span structures where critical loading events are typically caused by the cumulative effect of closely spaced vehicles with a high average load over the full loaded length. A high level of lateral load distribution is assumed with lane factors of 1.0 used for both lanes.

3.4 Baseline Simulation Method

Due to the large WIM dataset available, the baseline simulation uses a bootstrapping approach where random vehicles are repeatedly drawn from the observed data [42], as an alternative to Monte Carlo type simulations where samples are generated from statistical distributions of vehicle characteristics [43]. The vehicles are drawn from the time period of the congested event, and are assigned a lane of travel in proportion to the observed data. Vehicle gap and lane changing models are then applied as per the methodology used in the LSSM to transform free-flow WIM to congested conditions.

4 RESULTS

The daily maximum values for each simulation type are plotted in Figure 11 on a Gumbel probability paper plot [44]. Extreme values are extrapolated by means of a linear fit to the top $2\sqrt{n}$ data points [37, 45]. To allow potential comparison to Eurocode and AASHTO codes, lines corresponding to the 1000-year and 75-year return periods are added. For the assumed 250 congested workdays per year the probability, p, that an event may occur once in the return period is $1 - 1/(4\times1000) = 0.999996$ (Eurocode) or $1 - 1/(4\times75) = 0.999947$ (AASHTO). The values of Standard Extremal Variate for these probabilities, $-\ln(-\ln(p))$, are approximately 12.43 and 9.84 respectively.

The 1000-year load intensities for the different simulations methods are summarised in Table 1. The baseline random sampling method underestimates the measured load effect by 19.8%. The method does not allow for the inherent correlations present in the traffic data, which are clearly significant for long span load effects. The base LSSM applies no correlation between successive scenarios. While it provides for a better representation of the traffic patterns, it underestimates the measured load effect by 13.0%. Modelling the correlation between successive scenarios (LSSM incl correlation), while not providing a good fit to lighter load events, significantly improves the fit to the heavier load effects which influence the extrapolation and provides for an excellent match to the extreme value from the measured data (negligible difference).

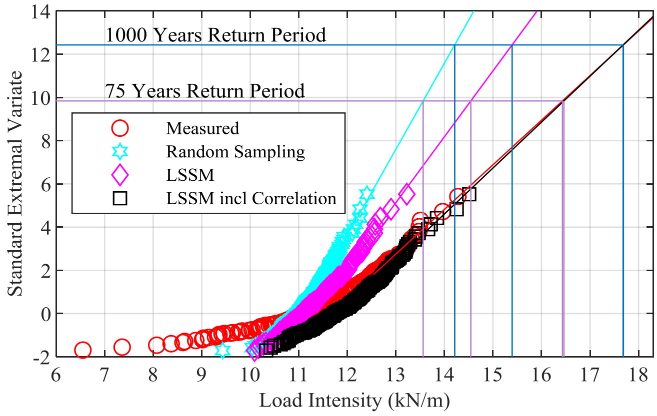

Figure 11 Extrapolation of daily maxim load effects to 75- and 1000-year return periods for each simulation method

Table 1 Comparison of 1000-year load intensities for different simulations methods

Simulation Method	**Load Intensity (Lane 1 + Lane 2) (kN/m)**	**% Difference**
Measured	17.7	-
Random Sampling	14.2	-19.8%
LSSM	15.4	-13.0%
LSSM incl Correlation	17.7	-0.0%

5 CONCLUSIONS

This paper details a method of simulating load effects on long span bridges termed 'Long Span Scenario Modelling' (LSSM). Free-flowing Weigh-in-Motion (WIM) data is converted into a congested traffic stream using lane changing and gap distribution models. The 'scenarios' are blocks of vehicles extracted from this stream which contain the inherent correlations between vehicle weights and positions. The correlation in load intensity between successive scenarios is explicitly modelled. The scenarios are used to simulate congested conditions for the required number of congestion events. The maximum load effect is calculated for each event and the results extrapolated for the required return period.

A large WIM dataset from a site in the USA is used to demonstrate the process. Recurring rush hour type congestion is simulated for 1 hour per day for 250 workdays. The total load effect for a 1000 m loaded length is determined for 75- and 1000-year return periods.

The LSSM is shown to better represent the long span load effect when compared to the measured traffic, particularly when the correlation between successive scenarios is accounted for. By modelling the correlations inherent in the measured traffic, the method ensures that combinations of heavy vehicles are simulated which have a greater influence on long span bridges than individual heavy vehicles.

The method can be extended to generate unobserved scenarios by introducing variations in the parameters. This would allow its use in cases where the WIM dataset is small or to undertake long run simulations. The method could also be combined with images of congested traffic, thereby removing uncertainties in converting WIM data from free-flowing to congested streams.

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