Acceleration based bridge weigh-in-motion using moving force identification

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ABSTRACT: Measuring vehicle axle weights is an important part of the monitoring of traffic. Traditional methods of measuring the static weights of vehicles at a weigh station is disruptive to drivers and time consuming for the authorities. It is more cost effective and efficient to directly measure moving vehicle weights either through pavement or bridge weigh-in-motion (WIM). The Bridge WIM concept, first proposed by Moses (1979), is an algorithm that uses measured bridge structural response to passing vehicles, to infer the weight of the passing vehicle and its axles. Moving Force identification (MFI) is, in effect, a dynamic Bridge WIM algorithm that gives a force time history, as a vehicle passes over the bridge. Previous MFI methods assumed the availability of measured deflection. However, deflection measurement requires a reference point, typically located at the underside of the bridge, something that is often not available. This creates difficulties and possible safety issues, particularly when a bridge is crossing a river or other infrastructure. In contrast, bridge accelerations can be measured using accelerometers placed on, for example, the handrail of the bridge, which is safer and easier to install. In this paper, a new MFI algorithm is proposed that uses, for the first time, measured bridge acceleration data as the input to compute vehicle axle weight. In our approach, first order regularization is used in the algorithm to improve the accuracy of the result. Numerical simulations demonstrate the MFI method's accuracy in estimating vehicle and axle weights.

KEY WORDS: Weigh-in-motion, WIM, moving force identification, MFI, acceleration, bridge, dynamics.

1. INTRODUCTION

As overweight vehicles can cause many problems such as pavement fatigue damage and increased risk of bridge overload. It follows that vehicle weight information is important for infrastructure management. The most accurate way to measure vehicle weights is at a static weigh station. However, this method requires vehicles to divert to a static measurement facility where they stop on the weigh scale or drive slowly over it. Drivers of overloaded vehicles will attempt to avoid these weigh stations, which results in a bias in the measured data. These issues are addressed in dynamic vehicle weighing methods known as weigh in motion (WIM). Moses[1] proposes a method that using bridge strain measurements to calculate gross vehicle weight, named bridge weigh in motion (BWIM). This algorithm has been extended to use other bridge measurements such as rotation and deflection. There are many BWIM systems in use today with good accuracy [2-4]. BWIM seeks to find the static axle weights of passing vehicles using the equations that relate applied loads to strains. An advance on this, known as moving force identification (MFI), uses a system of dynamic equations to find the applied axle force histories during the crossing event. The first research on MFI was carried by O'Connor and Chan in 1990 [5]. A popular MFI algorithm derives forces from the dynamic equations of motion, in state space form. This form is proposed by Trujillo [6] and is applied to the bridge case by Law and Fang in 2001 [7]. González et al [8] further develops this by regularizing the first derivatives of forces by Tikhonov regularization, making the solution more stable and smoother.

Most common BWIM and MFI algorithms use strain or deflection as inputs. Bridge accelerations are easier to measure than strain or deflection. For example, it can be measured using an accelerometer just placed on the bridge surface, but strain sensors need be attached to the bridge soffit. To date, acceleration is not generally used in BWIM or MFI algorithms. Some researchers do use accelerations but only to integrate and estimate deflection [9-12]. The issue with this approach is the initial condition problem introduced by Park[13]. If a wrong assumption is made in bridge initial deflection, it will cause significant error in the inferred deflection. In a recent work, O'Brien et al.[14] introduce a new static acceleration influence line concept and use it in a BWIM system. The results show great differences between real GVW and inferred GVW. Law's MFI algorithm[7] can use accelerations directly but bending moment is required as well. This paper proposes a new acceleration-based MFI algorithm that can use acceleration measured at one position to calculate a vehicle's dynamic forces during its passage over a bridge. Numerical simulation results show that good accuracy can be achieved in the calculation of vehicle forces.

2. MOVING FORCE IDENTIFICATION ALGORITHM USING ACCELERATION

As introduced in the last section, the MFI algorithm is derived from the equations of motion for the bridge:

$$M_g a + C_g v + K_g u = [L]g(t) \tag{1}$$

where M_g, C_g, K_g are the bridge mass, damping, and stiffness matrices of size $[n_{dof} \times n_{dof}]$. n_{dof} is the number of degrees of freedom in the bridge model. [L] is the location matrix that refers to the position of the vehicle axles. The size of the location matrix is $[n_{dof} \times n_{axles}]$ where n_{axles} denotes the number of vehicle axles. The vectors, a, v and u denote bridge element nodal accelerations, velocities and displacements. The function, g(t) is the force matrix of size $[n_{dof} \times 1]$.

The state space form of Eq. 1 is written as,

$$\left\{\frac{dX}{dt}\right\} = [A]\{X\} + [B]g(t) \tag{2}$$

where X is the vector containing nodal displacement and velocity:

$$\{X\} = \begin{Bmatrix} u \\ v \end{Bmatrix} \tag{3}$$

Matrices A and B are,

$$[A] = \begin{bmatrix} 0 & I \\ -{M_g}^{-1}K_g & -{M_g}^{-1}C_g \end{bmatrix} \quad (4)$$

$$[B] = \begin{bmatrix} 0 \\ {M_g}^{-1}[L] \end{bmatrix} \quad (5)$$

Trujillo [6] gives the solution as:

$$\{X\}_{j+1} = M\{X\}_j + P\{g\}_j \quad (6)$$

where:

$$M = \left[e^{[A]h}\right] \quad (7)$$

$$P = \left[\left[e^{[A]h} - I\right][A]^{-1}\begin{pmatrix} 0 \\ \left[M_g\right]^{-1}[L] \end{pmatrix}\right] \quad (8)$$

Gonzalez et al.[8] regularize the first derivative of the force, giving,

$$\begin{Bmatrix} X \\ g \end{Bmatrix}_{j+1} = \begin{bmatrix} M & P_j \\ 0 & I \end{bmatrix}\begin{Bmatrix} X \\ g \end{Bmatrix}_j + \begin{Bmatrix} 0 \\ I \end{Bmatrix}\{r\}_j \quad (9)$$

where r_j is the first derivative of force. The error function is defined as the minimum of the squared differences between inferred measurement, X and actual, measurement d_j. Tikhonov regularization is used in this error function with parameter B. The error function at any time step, j is written as:

$$E(X_j, r_j) = \min\left[\left(X_j - d_j, W(X_j - d_j)\right) + \left(r_j, Br_j\right)\right] = f_j(X_j) \quad (10)$$

The matrix, W is the identity matrix. r can be obtained by setting the first-order partial derivatives of this function to zero.

By regularizing the first derivative of force, the stability of the force solution has been greatly improved, especially at the beginning and end of the calculation process. This method can use displacement and velocity as input as the vector, X contains that information. However, acceleration is not contained in vector X, so it cannot be used directly. Now, we will give a way to incorporate acceleration into this algorithm. When the scan rate, Δt is small enough, the derivative of the vector, X can be written as:

$$\{\dot{X}\}_{j+1} = \frac{1}{\Delta t}\left(\{X\}_{j+1} - \{X\}_j\right) \quad (11)$$

Substituting Eq. 11 into Eq. 6 gives:

$$\{\dot{X}\}_{j+1} = \frac{([\mathrm{M}]\text{-}\mathrm{I})}{\Delta \mathrm{t}}\{\mathrm{X}\}_\mathrm{j} + \frac{[\mathrm{P}]}{\Delta \mathrm{t}}\{\mathrm{g}\}_\mathrm{j} \quad (12)$$

Consider a new vector,

$$\{\tilde{X}\}_{j+1} = \begin{Bmatrix} X \\ \dot{X} \\ g \end{Bmatrix}_{j+1} \quad (13)$$

Combining these equations gives:

$$\{\tilde{X}\}_{j+1} = \widetilde{M}\{\tilde{X}\}_j + T\{r\}_j \quad (14)$$

where

$$\widetilde{M} = \begin{bmatrix} M & 0 & P_j \\ \frac{([\mathrm{M}]\text{-}\mathrm{I})}{\Delta \mathrm{t}} & 0 & \frac{[P_j]}{\Delta \mathrm{t}} \\ 0 & 0 & I \end{bmatrix} \quad (15)$$

$$T = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \quad (16)$$

3. NUMERICAL MODEL

A Finite Element bridge model and a half car model are used in this paper. A 16 m simply supported beam-and-slab bridge is simulated. The bridge cross section is assumed to be made up of Y3 beams topped by a 200 mm deep slab. A total of 16 bridge finite elements are proposed with 17 nodes in a simple beam model – Figure 1. Each node has two degrees of freedom, vertical displacement and rotation. Acceleration is sampled from the centre node in the vertical direction. Rayleigh damping is assumed to form damping matrix[15]. Table 1 shows the properties of the bridge model.

Table 1. Properties of bridge model

Span	16m
Number of finite elements	16
Total degrees of freedom	34
Young's Modulus	3.5×10^9 N/m^2
Cross sectional area	2.41 m^2
Second moment of area	0.289 m^4
Damping	3%

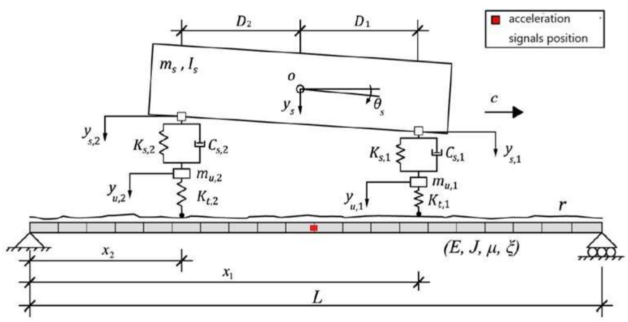

Figure 1. Coupled system developed form.

The half car model is illustrated in Figure 1. This kind of vehicle model contains 4 degrees of freedom: main body mass displacement (y_s) and rotation (θ_s) and axle mass displacements ($y_{u,1}$ and $y_{u,2}$). The body mass connects with each axle mass by a spring and a viscoelastic damper. The linear stiffnesses of the springs are, $K_{s,1}$ and $K_{t,2}$ and the damping coefficients are, $C_{s,1}$ and $C_{t,2}$. Two springs under the axle masses represent the tyres, with linear stiffnesses, $K_{t,1}$ *and* $K_{t,2.}$ The distance between axles and the centre of gravity are D_1 and D_2 respectively. Table 2 shows the values for each parameter used in the simulation.

Table 2. Half car model properties.

Property	Symbol/Units	Value
Body mass	m_s (kg)	*11200*
Axle mass	$M_{u,1}$ (kg)	*1100*
	$M_{u,2}$ (kg)	*700*
Suspension stiffness	$K_{s,1}$ (N/m)	$0.4x10^6$
	$K_{s,2}$ (N/m)	$1x10^6$
Suspension damping	$C_{s,1}$ (N/m)	$10x10^3$
	$C_{s,2}$ (N/m)	$20x10^3$
Tyre stiffness	$K_{t,1}$ (N/m)	$1.75x10^6$
	$K_{t,2}$ (N/m)	$3x10^6$
Axle spacing	D_1 & $D2$ (m)	*2.5*

4. NUMERICAL SIMULATION RESULT

This section shows a typical MFI algorithm result using mid-span acceleration. The mid-span bridge acceleration is gathered for the case of a 12-t half car passing over the bridge at 25m/s. A Class A road profile is generated in accordance with the ISO standard [16]. Acceleration is sampled at a 5000 Hz scan rate. Firstly, the maximum curvature in the L curve is obtained to get the optimal regularization parameter, B = 10^{-10}.

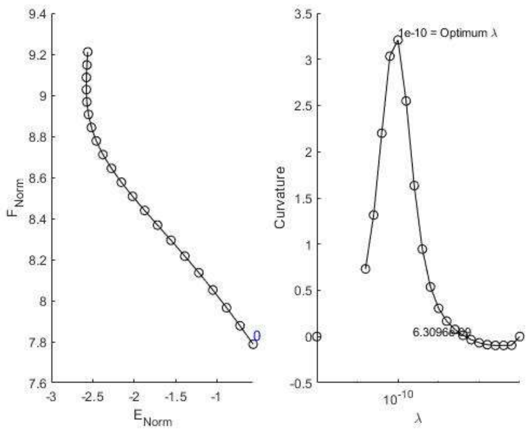

Figure 2. Regularization: a) L curve, b) curvature of L curve.

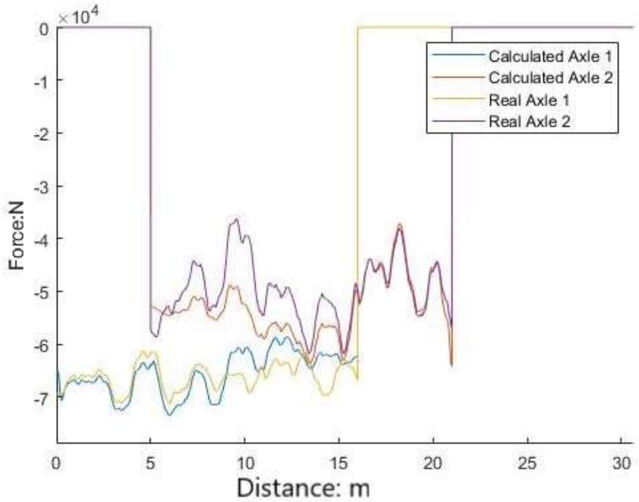

Figure 3. MFI result of dynamic axles force.

The optimal value of B is used in the MFI algorithm to get the force history and is presented in Figure 3. The vehicle is 5 m long and the bridge is located from $x = 0$ to 16 m in the figure. Hence, the first axle force history starts at $x = 0$ and the second at $x = 5$. The figure shows that, while there are significant differences, the calculated axle force histories are reasonable estimates of the actual real functions. From the result, it is concluded that using an accelerometer in one position is enough to estimate axle dynamic forces. When the second axle come onto the bridge, the difference between calculated and actual forces trends to increase. In this case, the second calculated axle force is greater than the real force most of the time. The average dynamic force is used to calculate vehicle gross vehicle weight (GVW) and gives an average of 12,098 kg. The real GVW is 12,000 kg. In this case, the difference between inferred and real GVW is 0.82%.

5. CONCLUSION

This paper proposes a new MFI algorithm that can use measured bridge acceleration as an input to calculate vehicle dynamic axle force. Compared with other MFI algorithms which use strain or deflection, acceleration can be directly used and only one accelerometer is enough to calculate vehicle dynamic axle forces. The simulation in MATLAB shows an acceptable level of accuracy in the calculation of GVW.

ACKNOWLEDGMENTS

The Author wishes to acknowledge the financial support received from University College Dublin and the Chinese Scholarship Council.

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